NCERT
Class 9th Maths
Chapter 1: Number Systems

Exercise 1.1

Question 1:

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Answer:

Yes. Zero is a rational number as it can be represented as $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ etc.

Question 2:

Find six rational numbers between 3 and 4.

Answer:

There are infinite rational numbers in between 3 and 4.

3 and 4 can be represented as $\frac{24}{8}$ and $\frac{32}{8}$ respectively.

Therefore, rational numbers between 3 and 4 are

$$\frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8}, \frac{29}{8}, \frac{30}{8}$$

Question 3:

Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Answer:

There are infinite rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

$$\frac{3}{5} = \frac{3\times6}{5\times6} = \frac{18}{30}$$

$$\frac{4}{5} = \frac{4\times6}{5\times6} = \frac{24}{30}$$

Therefore, rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are

$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

Question 4:

State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Answer:

(i) True; since the collection of whole numbers contains all natural numbers.

(ii) False; as integers may be negative but whole numbers are positive. For example: -3 is an integer but not a whole number.

(iii) False; as rational numbers may be fractional but whole numbers may not be. For example: $\frac{1}{5}$ is a rational number but not a whole number.

Exercise 1.2

Question 1:

State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Answer:

(i) True; since the collection of real numbers is made up of rational and irrational numbers.

(ii) False; as negative numbers cannot be expressed as the square root of any other number.

(iii) False; as real numbers include both rational and irrational numbers. Therefore, every real number cannot be an irrational number.

Question 2:

Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Answer:

If numbers such as $\sqrt{4}=2,\ \sqrt{9}=3$ are considered,

Then here, 2 and 3 are rational numbers. Thus, the square roots of all positive integers are not irrational.

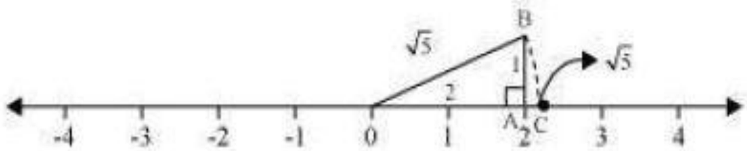
Question 3:

Show how $\sqrt{5}$ can be represented on the number line.

Answer:

We know that, $\sqrt{4}=2$

And, $\sqrt{5}=\sqrt{(2)^2+(1)^2}$



Mark a point 'A' representing 2 on number line. Now, construct AB of unit length perpendicular to OA. Then, taking O as centre and OB as radius, draw
an arc intersecting number line at C.

C is representing $\sqrt{5}$.

Exercise 1.3

Question 1:

Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Answer:

(i) $\frac{36}{100} = 0.36$

Terminating

(ii) $\frac{1}{11} = 0.090909......$ $= 0.\overline{09}$

Non-terminating repeating

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$

Terminating

(iv) $\frac{3}{13} = 0.230769230769....$ $= 0.\overline{230769}$

Non-terminating repeating

(v) $\frac{2}{11} = 0.18181818........$ $= 0.\overline{18}$

Non-terminating repeating

(vi) $\frac{329}{400} = 0.8225$

Terminating

Question 2:

You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[**Hint:** Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Answer:

Yes. It can be done as follows.

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$$

Question 3:

Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Answer:

(i) $0.\overline{6} = 0.666...$

Let x = 0.666...

$10x$ = 6.666...

$10x = 6 + x$

$9x = 6$

$x = \frac{2}{3}$

(ii) $0.4\overline{7} = 0.4777.....$

$= \frac{4}{10} + \frac{0.777}{10}$

Let $x = 0.777...$

$10x = 7.777...$

$10x = 7 + x$

$x = \frac{7}{9}$

$$\frac{4}{10} + \frac{0.777...}{10} = \frac{4}{10} + \frac{7}{90}$$
$$= \frac{36+7}{90} = \frac{43}{90}$$

(iii) $0.\overline{001} = 0.001001...$

Let $x = 0.001001...$

$1000x = 1.001001...$

$1000x = 1 + x$

$999x = 1$

$x = \frac{1}{999}$

Question 4:

Express 0.99999...in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer:

Let $x = 0.9999...$

$10x = 9.9999...$

$10x = 9 + x$

$9x = 9$

$x = 1$

Question 5:

What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Answer:

It can be observed that,

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$.

Question 6:

Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer:

Terminating decimal expansion will occur when denominator q of rational number $\frac{p}{q}$ is either of 2, 4, 5, 8, 10, and so on…

$$\frac{9}{4} = 2.25$$

$$\frac{11}{8} = 1.375$$

$$\frac{27}{5} = 5.4$$

It can be observed that terminating decimal may be obtained in the situation where prime factorisation of the denominator of the given fractions has the power of 2 only or 5 only or both.

Question 7:

Write three numbers whose decimal expansions are non-terminating non-recurring.

Answer:

3 numbers whose decimal expansions are non-terminating non-recurring are as follows.

0.505005000500005000005...

0.7207200720007200007200000...

0.08008000800008000008000008...

Question 8:

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Answer:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

3 irrational numbers are as follows.

0.73073007300073000073...

0.75075007500075000075...

0.79079007900079000079...

Question 9:

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 (v) 1.101001000100001...

(i) $\sqrt{23} = 4.79583152331\ \ldots$

As the decimal expansion of this number is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $\sqrt{225} = 15 = \frac{15}{1}$

It is a rational number as it can be represented in $\frac{p}{q}$ form.

(iii) 0.3796

As the decimal expansion of this number is terminating, therefore, it is a rational number.

(iv) 7.478478 ... $= 7.\overline{478}$

As the decimal expansion of this number is non-terminating recurring, therefore, it is a rational number.

(v) 1.10100100010000 ...

As the decimal expansion of this number is non-terminating non-repeating, therefore, it is an irrational number.

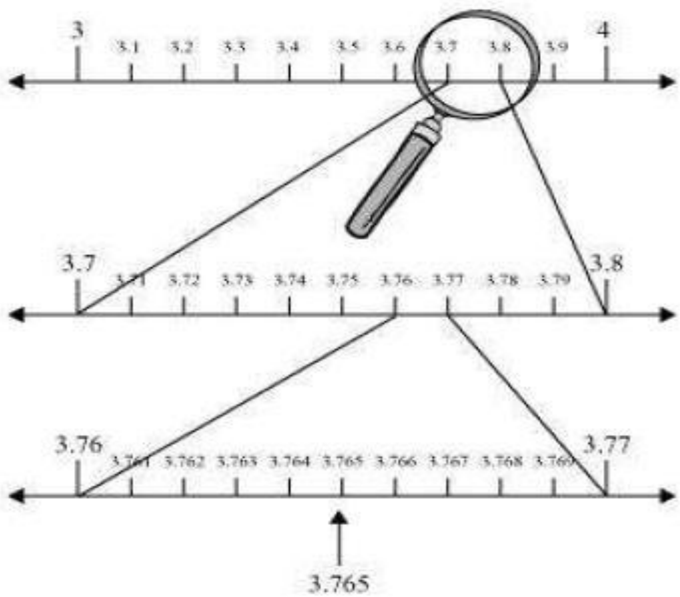
Exercise 1.4

Question 1:

Visualise 3.765 on the number line using successive magnification.

Answer:

3.765 can be visualised as in the following steps.



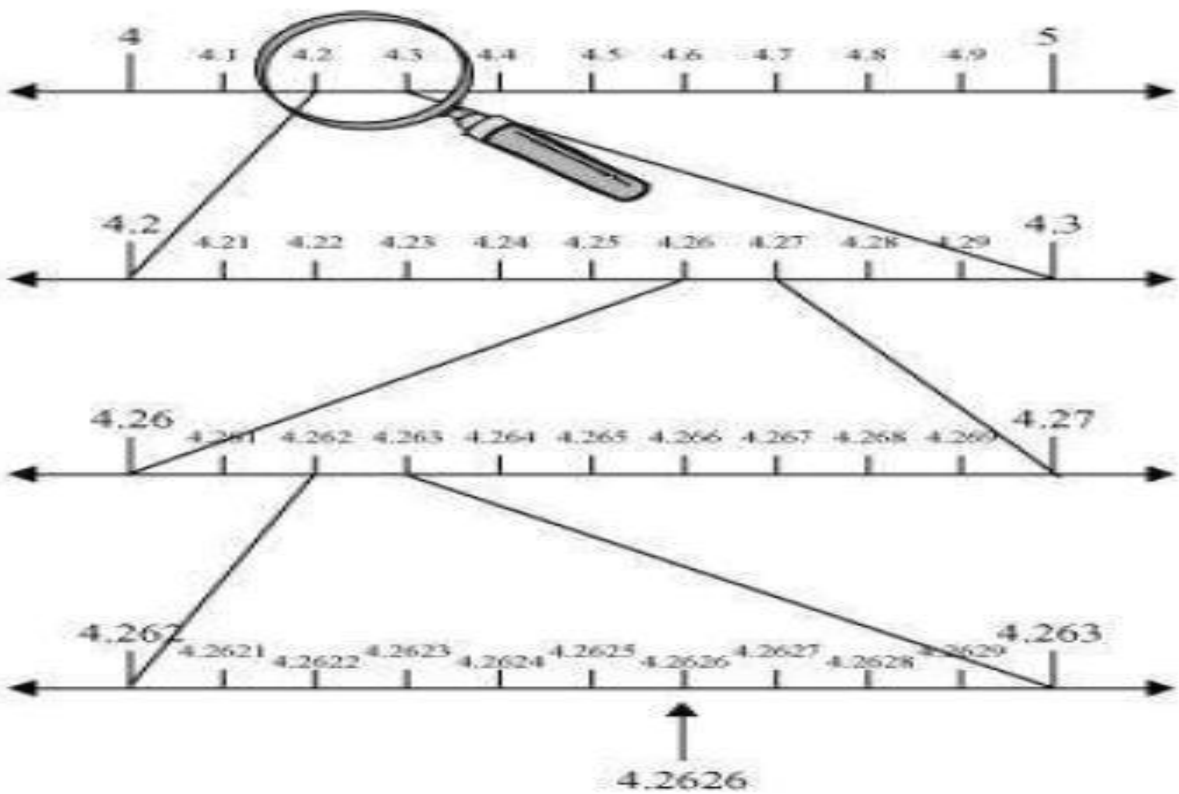
Question 2:

Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Answer:

$4.\overline{26}$ = 4.2626...

4.2626 can be visualised as in the following steps.


4
4.1
4.2
4.3
4.4
4.5
4.6
4.7
4.8
4.9
5
4.2
4.21
4.22
4.23
4.24
4.25
4.26
4.27
4.28
4.29
4.3
4.26
4.262
4.263
4.264
4.265
4.266
4.267
4.268
4.27
4.262
4.2621
4.2622
4.2623
4.2624
4.2625
4.2626
4.2627
4.2628
4.263
4.2626

Exercise 1.5

Question 1:

1Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Answer:

(i) $2-\sqrt{5}$ = 2 − 2.2360679...

= − 0.2360679...

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}=3=\frac{3}{1}$

As it can be represented in $\frac{p}{q}$ form, therefore, it is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

As it can be represented in $\frac{p}{q}$ form, therefore, it is a rational number.

(iv) $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=0.7071067811...$

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

(v) 2π = 2(3.1415 ...)

= 6.2830 ...

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

Question 2:

Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$ (iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Answer:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$

$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=(3)^2-\left(\sqrt{3}\right)^2$

$= 9 - 3 = 6$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{2}\right)$

$=5+2+2\sqrt{10}=7+2\sqrt{10}$

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$

$= 5 - 2 = 3$

Question 3:

Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi=\frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer:

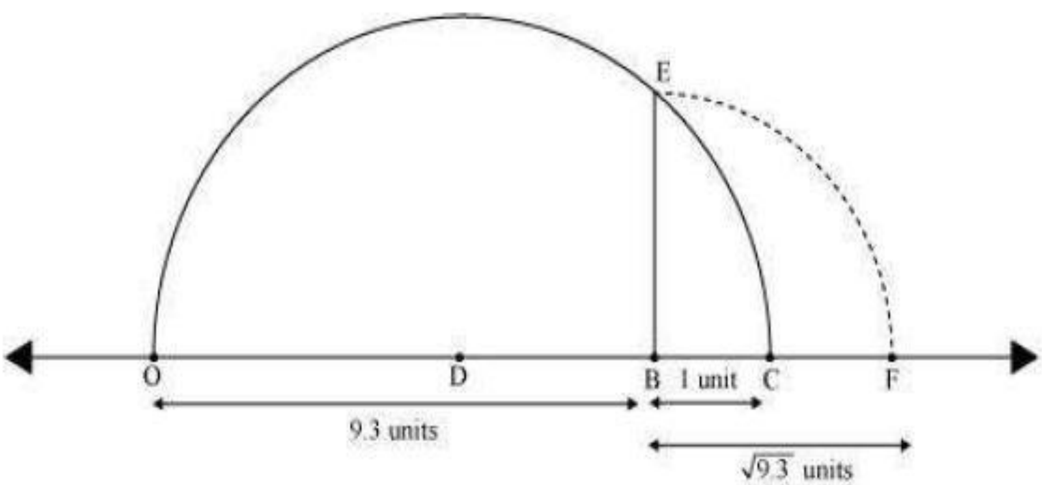
There is no contradiction. When we measure a length with scale or any other instrument, we only obtain an approximate rational value. We never obtain an exact value. For this reason, we may not realise that either c or d is irrational. Therefore, the fraction $\frac{c}{d}$ is irrational. Hence, π is irrational.

Question 4:

Represent $\sqrt{9.3}$ on the number line.

Answer:

Mark a line segment OB = 9.3 on number line. Further, take BC of 1 unit. Find the mid-point D of OC and draw a semi-circle on OC while taking D as its centre. Draw a perpendicular to line OC passing through point B. Let it intersect the semi-circle at E. Taking B as centre and BE as radius, draw an arc intersecting number line at F. BF is $\sqrt{9.3}$.



Question 5:

Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Answer:

(i) $\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{1\times\sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)}\left(\sqrt{7}+\sqrt{6}\right)$

$=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$

$=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\frac{\sqrt{7}+\sqrt{6}}{1}=\sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{1}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}\left(\sqrt{5}-\sqrt{2}\right)$

$=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{5}-\sqrt{2}}{5-2}$

$=\frac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\frac{1}{\sqrt{7}-2}=\frac{1}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}\left(\sqrt{7}+2\right)$

$=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$

$=\frac{\sqrt{7}+2}{7-4}=\frac{\sqrt{7}+2}{3}$

Exercise 1.6

Question 1:

Find:

(i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Answer:

(i)

$$64^{\frac{1}{2}} = \left(2^6\right)^{\frac{1}{2}}$$

$$= 2^{6\times\frac{1}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 2^3 = 8$$

(ii)

$$32^{\frac{1}{5}} = \left(2^5\right)^{\frac{1}{5}}$$

$$= (2)^{5\times\frac{1}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 2^1 = 2$$

(iii)

$$(125)^{\frac{1}{3}} = \left(5^3\right)^{\frac{1}{3}}$$

$$= 5^{3\times\frac{1}{3}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 5^1 = 5$$

Question 2:

Find:

(i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Answer:

(i)

$$9^{\frac{3}{2}} = \left(3^2\right)^{\frac{3}{2}}$$

$$= 3^{2\times\frac{3}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 3^3 = 27$$

(ii)

$$(32)^{\frac{2}{5}} = \left(2^5\right)^{\frac{2}{5}}$$

$$= 2^{5\times\frac{2}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 2^2 = 4$$

(iii)

$$(16)^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}}$$

$$= 2^{4\times\frac{3}{4}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 2^3 = 8$$

(iv)

$$(125)^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}} \qquad \left[a^{-m} = \frac{1}{a^m}\right]$$

$$= \frac{1}{\left(5^3\right)^{\frac{1}{3}}}$$

$$= \frac{1}{5^{3\times\frac{1}{3}}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= \frac{1}{5}$$

Question 3:

Simplify:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Answer:

(i)

$$2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} \qquad \left[a^m.a^n = a^{m+n}\right]$$

$$= 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$$

(ii)

$$\left(\frac{1}{3^3}\right)^7 = \frac{1}{3^{3\times7}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= \frac{1}{3^{21}}$$

$$= 3^{-21} \qquad \left[\frac{1}{a^m} = a^{-m}\right]$$

(iii)

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right]$$

$$= 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$$

(iv)

$$7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7\times8)^{\frac{1}{2}} \qquad [a^m.b^m = (ab)^m]$$

$$= (56)^{\frac{1}{2}}$$

NCERT
Class 9th Maths
Chapter 2: Polynomials

Exercise 2.1

Question 1:

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$ (v) $x^{10} + y^3 + t^{50}$

Answer:

(i) $4x^2 - 3x + 7$

Yes, this expression is a polynomial in one variable *x*.

(ii) $y^2 + \sqrt{2}$

Yes, this expression is a polynomial in one variable *y*.

(iii) $3\sqrt{t} + t\sqrt{2}$

No. It can be observed that the exponent of variable *t* in term $3\sqrt{t}$ is $\frac{1}{2}$, which is not a whole number. Therefore, this expression is not a polynomial.

(iv) $y + \frac{2}{y}$

No. It can be observed that the exponent of variable *y* in term $\frac{2}{y}$ is −1, which is not a whole number. Therefore, this expression is not a polynomial.

(v) $x^{10} + y^3 + t^{50}$

No. It can be observed that this expression is a polynomial in 3 variables *x*, *y*, and *t*. Therefore, it is not a polynomial in one variable.

Question 2:

Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$ (ii) $2-x^2+x^3$

(iii) $\frac{\pi}{2}x^2+x$ (iv) $\sqrt{2}x-1$

Answer:

(i) $2+x^2+x$

In the above expression, the coefficient of x^2 is 1.

(ii) $2-x^2+x^3$

In the above expression, the coefficient of x^2 is −1.

(iii) $\frac{\pi}{2}x^2+x$

In the above expression, the coefficient of x^2 is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1$, or

$0.x^2+\sqrt{2}x-1$

In the above expression, the coefficient of x^2 is 0.

Question 3:

Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

Binomial has two terms in it. Therefore, binomial of degree 35 can be written as $x^{35}+x^{34}$.

Monomial has only one term in it. Therefore, monomial of degree 100 can be written as x^{100}.

Question 4:

Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$

(iii) $5t - \sqrt{7}$ (iv) 3

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) $5x^3 + 4x^2 + 7x$

This is a polynomial in variable *x* and the highest power of variable *x* is 3. Therefore, the degree of this polynomial is 3.

(ii) $4 - y^2$

This is a polynomial in variable *y* and the highest power of variable *y* is 2. Therefore, the degree of this polynomial is 2.

(iii) $5t - \sqrt{7}$

This is a polynomial in variable *t* and the highest power of variable *t* is 1. Therefore, the degree of this polynomial is 1.

(iv) 3

This is a constant polynomial. Degree of a constant polynomial is always 0.

Question 5:

Classify the following as linear, quadratic and cubic polynomial:

(i) $x^2 + x$ (ii) $x - x^3$ (iii) $y + y^2 + 4$ (iv) $1 + x$ (v) $3t$

(vi) r^2 (vii) $7x^3$

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively.

(i) $x^2 + x$ is a quadratic polynomial as its degree is 2.

(ii) $x - x^3$ is a cubic polynomial as its degree is 3.

(iii) $y+y^2+4$ is a quadratic polynomial as its degree is 2.

(iv) 1 + x is a linear polynomial as its degree is 1.

(v) $3t$ is a linear polynomial as its degree is 1.

(vi) r^2 is a quadratic polynomial as its degree is 2.

(vii) $7x^3$ is a cubic polynomial as its degree is 3.

Exercise 2.2

Question 1:

Find the value of the polynomial $5x-4x^2+3$ at

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Answer:

(i) $p(x)=5x-4x^2+3$

$$p(0)=5(0)-4(0)^2+3$$
$$=3$$

(ii) $p(x)=5x-4x^2+3$

$$p(-1)=5(-1)-4(-1)^2+3$$
$$=-5-4(1)+3=-6$$

(iii) $p(x)=5x-4x^2+3$

$$p(2)=5(2)-4(2)^2+3$$
$$=10-16+3=-3$$

Question 2:

Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

Answer:

(i) $p(y) = y^2 - y + 1$

$p(0) = (0)^2 - (0) + 1 = 1$

$p(1) = (1)^2 - (1) + 1 = 1$

$p(2) = (2)^2 - (2) + 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$

$p(1) = 2 + (1) + 2(1)^2 - (1)^3$

$= 2 + 1 + 2 - 1 = 4$

$p(2) = 2 + 2 + 2(2)^2 - (2)^3$

$= 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$

$p(0) = (0)^3 = 0$

$p(1) = (1)^3 = 1$

$p(2) = (2)^3 = 8$

(iv) $p(x) = (x - 1)(x + 1)$

$p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$

$p(1) = (1 - 1)(1 + 1) = 0(2) = 0$

$p(2) = (2 - 1)(2 + 1) = 1(3) = 3$

Question 3:

Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1, x=-\frac{1}{3}$ (ii) $p(x)=5x-\pi, x=\frac{4}{5}$

(iii) $p(x) = x^2 - 1, x = 1, -1$ (iv) $p(x) = (x + 1)(x - 2), x = -1, 2$

(v) $p(x) = x^2, x = 0$ (vi) $p(x)=lm+m, x=-\frac{m}{l}$

(vii) $p(x)=3x^2-1, x=-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ (viii) $p(x)=2x+1, x=\frac{1}{2}$

Answer:

(i) If $x=\frac{-1}{3}$ is a zero of given polynomial $p(x) = 3x + 1$, then $p\left(-\frac{1}{3}\right)$ should be 0.

Here, $p\left(\frac{-1}{3}\right)=3\left(\frac{-1}{3}\right)+1=-1+1=0$

Therefore, $x=\frac{-1}{3}$ is a zero of the given polynomial.

(ii) If $x=\frac{4}{5}$ is a zero of polynomial $p(x) = 5x - \pi$, then $p\left(\frac{4}{5}\right)$ should be 0.

$$Here,\ p\left(\frac{4}{5}\right)=5\left(\frac{4}{5}\right)-\pi=4-\pi$$

$$As\ p\left(\frac{4}{5}\right)\neq 0,$$

Therefore, $x=\frac{4}{5}$ is not a zero of the given polynomial.

(iii) If x = 1 and x = −1 are zeroes of polynomial $p(x) = x^2 - 1$, then $p(1)$ and $p(-1)$ should be 0.

Here, $p(1) = (1)^2 - 1 = 0$, and

$p(-1) = (-1)^2 - 1 = 0$

Hence, x = 1 and −1 are zeroes of the given polynomial.

(iv) If x = −1 and x = 2 are zeroes of polynomial $p(x) = (x+1)(x-2)$, then $p(-1)$ and $p(2)$should be 0.

Here, $p(-1) = (-1+1)(-1-2) = 0(-3) = 0$, and

$p(2) = (2+1)(2-2) = 3(0) = 0$

Therefore, x = −1 and x = 2 are zeroes of the given polynomial.

(v) If x = 0 is a zero of polynomial $p(x) = x^2$, then $p(0)$ should be zero.

Here, $p(0) = (0)^2 = 0$

Hence, x = 0 is a zero of the given polynomial.

(vi) If $x=\frac{-m}{l}$ is a zero of polynomial $p(x) = lx + m$, then $p\left(\frac{-m}{l}\right)$ should be 0.

Here, $p\left(\frac{-m}{l}\right)=l\left(\frac{-m}{l}\right)+m=-m+m=0$

Therefore, $x=-\frac{m}{l}$ is a zero of the given polynomial.

(vii) If $x=\frac{-1}{\sqrt{3}}$ and $x=\frac{2}{\sqrt{3}}$ are zeroes of polynomial $p(x) = 3x^2 - 1$, then

$p\left(\frac{-1}{\sqrt{3}}\right)$ and $p\left(\frac{2}{\sqrt{3}}\right)$ should be 0.

Here, $p\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$, and

$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$

Hence, $x = \frac{-1}{\sqrt{3}}$ is a zero of the given polynomial. However, $x = \frac{2}{\sqrt{3}}$ is not a zero of the given polynomial.

(viii) If $x = \frac{1}{2}$ is a zero of polynomial $p(x) = 2x + 1$, then $p\left(\frac{1}{2}\right)$ should be 0.

Here, $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

As $p\left(\frac{1}{2}\right) \neq 0$,

Therefore, $x = \frac{1}{2}$ is not a zero of the given polynomial.

Question 4:

Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$ (ii) $p(x) = x - 5$ (iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$ (v) $p(x) = 3x$ (vi) $p(x) = ax$, $a \neq 0$

(vii) $p(x) = cx + d$, $c \neq 0$, c, are real numbers.

Answer:

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

(i) $p(x) = x + 5$

$p(x) = 0$

$x + 5 = 0$

$x = -5$

Therefore, for $x = -5$, the value of the polynomial is 0 and hence, $x = -5$ is a zero of the given polynomial.

(ii) $p(x) = x - 5$

$p(x) = 0$

$x - 5 = 0$

$x = 5$

Therefore, for $x = 5$, the value of the polynomial is0 and hence, $x = 5$ is a zero of the given polynomial.

(iii) $p(x) = 2x + 5$

$p(x) = 0$

$2x + 5 = 0$

$2x = -5$

$x = -\frac{5}{2}$

Therefore, for $x = -\frac{5}{2}$, the value of the polynomial is 0 and hence, $x = \frac{-5}{2}$ is a zero of the given polynomial.

(iv) $p(x) = 3x - 2$

$p(x) = 0$

$3x - 2 = 0$

$x = \frac{2}{3}$

Therefore, for $x = \frac{2}{3}$, the value of the polynomial is 0 and hence, $x = \frac{2}{3}$ is a zero of the given polynomial.

(v) $p(x) = 3x$

$p(x) = 0$

$3x = 0$

$x = 0$

Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.

(vi) $p(x) = ax$

$p(x) = 0$

$ax = 0$

$x = 0$

Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.

(vii) $p(x) = cx + d$

$p(x) = 0$

$cx + d = 0$

$$x = \frac{-d}{c}$$

Therefore, for $x = \frac{-d}{c}$, the value of the polynomial is 0 and hence, $x = \frac{-d}{c}$ is a zero of the given polynomial.

Exercise 2.3

Question 1:

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$ (ii) $x - \frac{1}{2}$ (iii) x

(iv) $x + \pi$ (v) $5 + 2x$

Answer:

(i) $x + 1$

By long division,

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
x+1 & x^3 + 3x^2 + 3x + 1 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & 2x^2 + 3x + 1 \\
 & 2x^2 + 2x \\
 & - \quad - \\
\hline
 & x + 1 \\
 & x + 1 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Therefore, the remainder is 0.

(ii) $x - \frac{1}{2}$

By long division,

$$
\begin{array}{r}
x^2+\frac{7}{2}x+\frac{19}{4} \\
x-\frac{1}{2}\overline{\smash{\big)}\;x^3+3x^2+3x+1} \\
x^3-\frac{x^2}{2} \\
-\quad + \\
\hline
\frac{7}{2}x^2+3x+1 \\
\frac{7}{2}x^2-\frac{7}{4}x \\
-\quad + \\
\hline
\frac{19}{4}x+1 \\
\frac{19}{4}x-\frac{19}{8} \\
-\quad + \\
\hline
\frac{27}{8} \\
\hline
\end{array}
$$

Therefore, the remainder is $\frac{27}{8}$.

(iii) x

By long division,

$$
\begin{array}{r|l}
 & x^2+3x+3 \\
\hline
x & x^3+3x^2+3x+1 \\
 & x^3 \\
 & - \\
\hline
 & 3x^2+3x+1 \\
 & 3x^2 \\
 & - \\
\hline
 & 3x+1 \\
 & 3x \\
 & - \\
\hline
 & 1 \\
\hline
\end{array}
$$

Therefore, the remainder is 1.

(iv) $x + \pi$

By long division,

$$
\begin{array}{r|l}
 & x^2+(3-\pi)x+(3-3\pi+\pi^2) \\
\hline
x+\pi & x^3+3x^2+3x+1 \\
 & x^3+\pi x^2 \\
 & - \quad - \\
\hline
 & (3-\pi)x^2+3x+1 \\
 & (3-\pi)x^2+(3-\pi)\pi x \\
 & - \quad\quad - \\
\hline
 & [3-3\pi+\pi^2]x+1 \\
 & [3-3\pi+\pi^2]x+(3-3\pi+\pi^2)\pi \\
 & - \quad\quad\quad - \\
\hline
 & [1-3\pi+3\pi^2-\pi^3] \\
\hline
\end{array}
$$

Therefore, the remainder is $-\pi^3+3\pi^2-3\pi+1$.

(v) $5 + 2x$

By long division,

$$\begin{array}{r|l} & \frac{x^2}{2}+\frac{x}{4}+\frac{7}{8} \\ \hline 2x+5 & x^3+3x^2+3x+1 \\ & x^3+\frac{5}{2}x^2 \\ & -\quad - \\ \hline & \frac{x^2}{2}+3x+1 \\ & \frac{x^2}{2}+\frac{5x}{4} \\ & -\quad - \\ \hline & \frac{7x}{4}+1 \\ & \frac{7}{4}x+\frac{35}{8} \\ & -\quad - \\ \hline & -\frac{27}{8} \\ \hline \end{array}$$

Therefore, the remainder is $-\frac{27}{8}$

Question 2:

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Answer:

By long division,

$$\begin{array}{r|l} & x^2+6 \\ \hline x-a & x^3-ax^2+6x-a \\ & x^3-ax^2 \\ & -\quad + \\ \hline & \qquad 6x-a \\ & \qquad 6x-6a \\ & \qquad -\quad + \\ \hline & \qquad\qquad 5a \\ \hline \end{array}$$

Therefore, when $x^3 - ax^2 + 6x - a$ is divided by $x - a$, the remainder obtained is $5a$.

Question 3:

Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Answer:

Let us divide $(3x^3 + 7x)$ by $(7 + 3x)$. If the remainder obtained is 0, then $7 + 3x$ will be a factor of $3x^3 + 7x$.

By long division,

$$\begin{array}{r|l} & x^2-\frac{7}{3}x+\frac{70}{9} \\ \hline 3x+7 & 3x^3+0x^2+7x \\ & 3x^3+7x^2 \\ & -\quad - \\ \hline & \quad -7x^2+7x \\ & \quad -7x^2-\frac{49x}{3} \\ & \quad +\qquad + \\ \hline & \qquad\quad \frac{70x}{3} \\ & \qquad\quad \frac{70x}{3}+\frac{490}{9} \\ & \qquad\quad -\qquad - \\ \hline & \qquad\qquad -\frac{490}{9} \\ \hline \end{array}$$

As the remainder is not zero, therefore, $7 + 3x$ is not a factor of $3x^3 + 7x$.

Exercise 2.4

Question 1:

Determine which of the following polynomials has $(x + 1)$ a factor:

(i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ (iv) $x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$

Answer:

(i) If $(x + 1)$ is a factor of $p(x) = x^3 + x^2 + x + 1$, then $p(-1)$ must be zero, otherwise $(x + 1)$ is not a factor of $p(x)$.

$p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 - 1 = 0$

Hence, $x + 1$ is a factor of this polynomial.

(ii) If $(x + 1)$ is a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$, then $p(-1)$ must be zero, otherwise $(x + 1)$ is not a factor of $p(x)$.

$p(x) = x^4 + x^3 + x^2 + x + 1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1 = 1$

As $p(-1) \neq 0$,

Therefore, $x + 1$ is not a factor of this polynomial.

(iii) If $(x + 1)$ is a factor of polynomial $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$, then $p(-1)$ must be 0, otherwise $(x + 1)$ is not a factor of this polynomial.

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

As $p(-1) \neq 0$,

Therefore, $x + 1$ is not a factor of this polynomial.

(iv) If$(x + 1)$ is a factor of polynomial $p(x) = x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$, then $p(-1)$ must be 0, otherwise $(x + 1)$ is not a factor of this polynomial.

$$p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$$
$$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$$
$$= 2\sqrt{2}$$

As $p(-1) \neq 0$,

Therefore, $(x + 1)$ is not a factor of this polynomial.

Question 2:

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Answer:

(i) If $g(x) = x + 1$ is a factor of the given polynomial $p(x)$, then $p(-1)$ must be zero.

$p(x) = 2x^3 + x^2 - 2x - 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= 2(-1) + 1 + 2 - 1 = 0$

Hence, $g(x) = x + 1$ is a factor of the given polynomial.

(ii) If $g(x) = x + 2$ is a factor of the given polynomial $p(x)$, then $p(-2)$ must be 0.

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -1$

As $p(-2) \neq 0$,

Hence, $g(x) = x + 2$ is not a factor of the given polynomial.

(iii) If $g(x) = x - 3$ is a factor of the given polynomial $p(x)$, then $p(3)$ must be 0.

$p(x) = x^3 - 4x^2 + x + 6$

$p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9 = 0$

Hence, $g(x) = x - 3$ is a factor of the given polynomial.

Question 3:

Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$ (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ (iv) $p(x) = kx^2 - 3x + k$

Answer:

If $x - 1$ is a factor of polynomial $p(x)$, then $p(1)$ must be 0.

(i) $p(x) = x^2 + x + k$

$p(1) = 0$

$\Rightarrow (1)^2 + 1 + k = 0$

$\Rightarrow 2 + k = 0$

$\Rightarrow k = -2$

Therefore, the value of k is -2.

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

$p(1) = 0$

$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$

$\Rightarrow 2 + k + \sqrt{2} = 0$

$\Rightarrow k = -2 - \sqrt{2} = -\left(2 + \sqrt{2}\right)$

Therefore, the value of k is $-\left(2 + \sqrt{2}\right)$.

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

$p(1) = 0$

$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k - \sqrt{2} + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$

Therefore, the value of k is $\sqrt{2} - 1$.

(iv) $p(x) = kx^2 - 3x + k$

$\Rightarrow p(1) = 0$

$\Rightarrow k(1)^2 - 3(1) + k = 0$

$\Rightarrow k - 3 + k = 0$

$\Rightarrow 2k - 3 = 0$

$\Rightarrow k = \frac{3}{2}$

Therefore, the value of k is $\frac{3}{2}$.

Question 4:

Factorise:

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$

Answer:

(i) $12x^2 - 7x + 1$

We can find two numbers such that $pq = 12 \times 1 = 12$ and $p + q = -7$. They are $p = -4$ and $q = -3$.

Here, $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1)$

(ii) $2x^2 + 7x + 3$

We can find two numbers such that $pq = 2 \times 3 = 6$ and $p + q = 7$.

They are $p = 6$ and $q = 1$.

Here, $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(iii) $6x^2 + 5x - 6$

We can find two numbers such that $pq = -36$ and $p + q = 5$.

They are $p = 9$ and $q = -4$.

Here,

$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

(iv) $3x^2 - x - 4$

We can find two numbers such that $pq = 3 \times (-4) = -12$

and $p + q = -1$.

They are $p = -4$ and $q = 3$.

Here,

$3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

Question 5:

Factorise:

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 + 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$

Answer:

(i) Let $p(x) = x^3 - 2x^2 - x + 2$

All the factors of 2 have to be considered. These are ± 1, ± 2.

By trial method,

$p(2) = (2)^3 - 2(2)^2 - 2 + 2$

$= 8 - 8 - 2 + 2 = 0$

Therefore, $(x - 2)$ is factor of polynomial $p(x)$.

Let us find the quotient on dividing $x^3 - 2x^2 - x + 2$ by $x - 2$.

By long division,

$$
\begin{array}{r}
x^2 - 3x + 2 \\
x+1 \overline{) \; x^3 - 2x^2 - x + 2} \\
x^3 + x^2 \quad\quad\quad \\
- \quad - \quad\quad\quad\quad \\
\hline
-3x^2 - x + 2 \\
-3x^2 - 3x \quad \\
+ \quad\; + \quad\quad \\
\hline
2x + 2 \\
2x + 2 \\
- \quad - \\
\hline
0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder

$\therefore x^3 - 2x^2 - x + 2 = (x + 1)(x^2 - 3x + 2) + 0$

$= (x + 1)[x^2 - 2x - x + 2]$

$= (x + 1)[x(x - 2) - 1(x - 2)]$

$= (x + 1)(x - 1)(x - 2)$

$= (x - 2)(x - 1)(x + 1)$

(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$

All the factors of 5 have to be considered. These are ± 1, ± 5.

By trial method,

$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5 = 0$

Therefore, $x + 1$ is a factor of this polynomial.

Let us find the quotient on dividing $x^3 + 3x^2 - 9x - 5$ by $x + 1$.

By long division,

$$
\begin{array}{l}
\qquad x^2-4x-5 \\
x+1\overline{\big)x^3-3x^2-9x-5} \\
\qquad x^3+\ x^2 \\
\quad\ -\qquad - \\
\hline
\qquad\quad -4x^2-9x-5 \\
\qquad\quad -4x^2-4x \\
\qquad\quad\ +\qquad + \\
\hline
\qquad\qquad\qquad -5x-5 \\
\qquad\qquad\qquad -5x-5 \\
\qquad\qquad\qquad\ +\qquad + \\
\hline
\qquad\qquad\qquad\qquad\quad 0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder

$\therefore x^3 - 3x^2 - 9x - 5 = (x + 1)\ (x^2 - 4x - 5) + 0$

$= (x + 1)\ (x^2 - 5x + x - 5)$

$= (x + 1)\ [(x\ (x - 5) + 1\ (x - 5)]$

$= (x + 1)\ (x - 5)\ (x + 1)$

$= (x - 5)\ (x + 1)\ (x + 1)$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

All the factors of 20 have to be considered. Some of them are ±1, ± 2, ± 4, ± 5 ……

By trial method,

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33 = 0$

As $p(-1)$ is zero, therefore, $x + 1$ is a factor of this polynomial $p(x)$.

Let us find the quotient on dividing $x^3 + 13x^2 + 32x + 20$ by $(x + 1)$.

By long division,

$$
\begin{array}{r}
x^2+12x+20 \\
x+1\overline{)x^3+13x^2+32x+20} \\
x^3+x^2 \\
-\quad - \\
\hline
12x^2+32x \\
12x^2+12x \\
-\quad - \\
\hline
20x+20 \\
20x+20 \\
-\quad - \\
\hline
0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder

$x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20) + 0$

$= (x + 1)(x^2 + 10x + 2x + 20)$

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 10)(x + 2)$

$= (x + 1)(x + 2)(x + 10)$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$

By trial method,

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1 = 0$

Therefore, $y - 1$ is a factor of this polynomial.

Let us find the quotient on dividing $2y^3 + y^2 - 2y - 1$ by $y - 1$.

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y - 1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & 3y^2 - 2y - 1 \\
 & 3y^2 - 3y \\
 & - \quad + \\
\hline
 & y - 1 \\
 & y - 1 \\
\hline
 & 0 \\
\hline
\end{array}
$$

$p(y) = 2y^3 + y^2 - 2y - 1$

$= (y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)[2y(y + 1) + 1(y + 1)]$

$= (y - 1)(y + 1)(2y + 1)$

Exercise 2.5

Question 1:

Use suitable identities to find the following products:

(i) $(x+4)(x+10)$ (ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$ (iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Answer:

(i) By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$,

$$(x+4)(x+10)=x^2+(4+10)x+4\times10$$
$$=x^2+14x+40$$

(ii) By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$,

$$(x+8)(x-10)=x^2+(8-10)x+(8)(-10)$$
$$=x^2-2x-80$$

(iii) $(3x+4)(3x-5)=9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)$

By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$,

$$9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)=9\left[x^2+\left(\frac{4}{3}-\frac{5}{3}\right)x+\left(\frac{4}{3}\right)\left(-\frac{5}{3}\right)\right]$$
$$=9\left[x^2-\frac{1}{3}x-\frac{20}{9}\right]$$
$$=9x^2-3x-20$$

(iv) By using the identity $(x+y)(x-y)=x^2-y^2$,

$$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=\left(y^2\right)^2-\left(\frac{3}{2}\right)^2$$
$$=y^4-\frac{9}{4}$$

(v) By using the identity $(x+y)(x-y)=x^2-y^2$,

$$(3-2x)(3+2x)=(3)^2-(2x)^2$$
$$=9-4x^2$$

Question 2:

Evaluate the following products without multiplying directly:

(i) 103 × 107 (ii) 95 × 96 (iii) 104 × 96

Answer:

(i) 103 × 107 = (100 + 3) (100 + 7)

= $(100)^2$ + (3 + 7) 100 + (3) (7)

[By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$, where x = 100, a = 3, and b = 7]

= 10000 + 1000 + 21

= 11021

(ii) 95 × 96 = (100 − 5) (100 − 4)

= $(100)^2$ + (− 5 − 4) 100 + (− 5) (− 4)

[By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$, where x = 100, a = −5, and b = −4]

= 10000 − 900 + 20

= 9120

(iii) 104 × 96 = (100 + 4) (100 − 4)

= $(100)^2 - (4)^2$ $\left[(x+y)(x-y)=x^2-y^2\right]$

= 10000 − 16

= 9984

Question 3:

Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Answer:

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x+y)(3x+y)$ $\quad\left[x^2 + 2xy + y^2 = (x+y)^2\right]$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y-1)(2y-1)$ $\quad\left[x^2 - 2xy + y^2 = (x-y)^2\right]$

(iii) $x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2$

$= \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$ $\quad\left[x^2 - y^2 = (x+y)(x-y)\right]$

Question 4:

Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$ (ii) $(2x-y+z)^2$

(iii) $(-2x+3y+2z)^2$ (iv) $(3a-7b-c)^2$

(v) $(-2x+5y-3z)^2$ (vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Answer:

It is known that,

$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i) $(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+2(x)(2y)+2(2y)(4z)+2(4z)(x)$

$= x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2 = (2x)^2+(-y)^2+(z)^2+2(2x)(-y)+2(-y)(z)+2(z)(2x)$

$= 4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

$= (-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(2z)(-2x)$

$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a-7b-c)^2$

$= (3a)^2+(-7b)^2+(-c)^2+2(3a)(-7b)+2(-7b)(-c)+2(-c)(3a)$

$= 9a^2+49b^2+c^2-42ab+14bc-6ac$

(v) $(-2x+5y-3z)^2$

$= (-2x)^2+(5y)^2+(-3z)^2+2(-2x)(5y)+2(5y)(-3z)+2(-3z)(-2x)$

$= 4x^2+25y^2+9z^2-20xy-30yz+12xz$

(vi) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$

$= \left(\frac{1}{4}a\right)^2+\left(-\frac{1}{2}b\right)^2+(1)^2+2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)+2\left(-\frac{1}{2}b\right)(1)+2\left(\frac{1}{4}a\right)(1)$

$= \frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$

Question 5:

Factorise:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Answer:

It is known that,

$$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

$$=(2x)^2+(3y)^2+(-4z)^2+2(2x)(3y)+2(3y)(-4z)+2(2x)(-4z)$$
$$=(2x+3y-4z)^2$$
$$=(2x+3y-4z)(2x+3y-4z)$$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$$=\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\left(-\sqrt{2}x\right)(y)+2(y)\left(2\sqrt{2}z\right)+2\left(-\sqrt{2}x\right)\left(2\sqrt{2}z\right)$$
$$=\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$$
$$=\left(-\sqrt{2}x+y+2\sqrt{2}z\right)\left(-\sqrt{2}x+y+2\sqrt{2}z\right)$$

Question 6:

Write the following cubes in expanded form:

(i) $(2x+1)^3$ (ii) $(2a-3b)^3$

(iii) $\left[\frac{3}{2}x+1\right]^3$ (iv) $\left[x-\frac{2}{3}y\right]^3$

Answer:

It is known that,

$$(a+b)^3 = a^3+b^3+3ab(a+b)$$

$$\text{and}\,(a-b)^3 = a^3-b^3-3ab(a-b)$$

(i) $(2x+1)^3 = (2x)^3+(1)^3+3(2x)(1)(2x+1)$

$= 8x^3 + 1 + 6x(2x+1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a-3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$

$= 8a^3 - 27b^3 - 18ab(2a-3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $\left[\frac{3}{2}x+1\right]^3 = \left[\frac{3}{2}x\right]^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$

$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(vi) $\left[x-\frac{2}{3}y\right]^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

Question 7:

Evaluate the following using suitable identities:

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

Answer:

It is known that,

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

and $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(i) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - 300(99)$

$= 1000000 - 1 - 29700$

$= 970299$

(ii) $(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

$= 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200$

$= 1061208$

(iii) $(998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$

$= 1000000000 - 8 - 6000(998)$

$= 1000000000 - 8 - 5988000$

$= 1000000000 - 5988008$

$= 994011992$

Question 8:

Factorise each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Answer:

It is known that,

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

and $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3(2a)^2 b + 3(2a)(b)^2$

$= (2a+b)^3$

$= (2a+b)(2a+b)(2a+b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 3(2a)^2 b + 3(2a)(b)^2$

$= (2a-b)^3$

$= (2a-b)(2a-b)(2a-b)$

(iii) $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3(3)^2(5a) + 3(3)(5a)^2$

$= (3-5a)^3$

$= (3-5a)(3-5a)(3-5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$

$= (4a-3b)^3$

$= (4a-3b)(4a-3b)(4a-3b)$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2\left(\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2$

$= \left(3p - \frac{1}{6}\right)^3$

$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

Question 9:

Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Answer:

(i) It is known that,

$$(x+y)^3=x^3+y^3+3xy(x+y)$$

$$\begin{aligned}x^3+y^3&=(x+y)^3-3xy(x+y)\\&=(x+y)\left[(x+y)^2-3xy\right]\\&=(x+y)(x^2+y^2+2xy-3xy)\\&=(x+y)(x^2+y^2-xy)\\&=(x+y)(x^2-xy+y^2)\end{aligned}$$

(ii) It is known that,

$$(x-y)^3=x^3-y^3-3xy(x-y)$$

$$\begin{aligned}x^3-y^3&=(x-y)^3+3xy(x-y)\\&=(x-y)\left[(x-y)^2+3xy\right]\\&=(x-y)(x^2+y^2-2xy+3xy)\\&=(x-y)(x^2+y^2+xy)\\&=(x-y)(x^2+xy+y^2)\end{aligned}$$

Question 10:

Factorise each of the following:

(i) $27y^3+125z^3$

(ii) $64m^3-343n^3$

[**Hint:** See question 9.]

Answer:

(i) $27y^3 + 125z^3$

$= (3y)^3 + (5z)^3$

$= (3y+5z)\left[(3y)^2 + (5z)^2 - (3y)(5z)\right] \quad \left[\because a^3 + b^3 = (a+b)(a^2+b^2-ab)\right]$

$= (3y+5z)\left[9y^2 + 25z^2 - 15yz\right]$

(ii) $64m^3 - 343n^3$

$= (4m)^3 - (7n)^3$

$= (4m-7n)\left[(4m)^2 + (7n)^2 + (4m)(7n)\right] \quad \left[\because a^3 - b^3 = (a-b)(a^2+b^2+ab)\right]$

$= (4m-7n)\left[16m^2 + 49n^2 + 28mn\right]$

Question 11:

Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Answer:

It is known that,

$x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\therefore 27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x+y+z)\left[(3x)^2 + y^2 + z^2 - (3x)(y) - (y)(z) - z(3x)\right]$

$= (3x+y+z)\left[9x^2 + y^2 + z^2 - 3xy - yz - 3xz\right]$

Question 12:

Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]$

Answer:

It is known that,

$x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$$= \frac{1}{2}(x+y+z)\left[2x^2+2y^2+2z^2-2xy-2yz-2zx\right]$$

$$= \frac{1}{2}(x+y+z)\left[\left(x^2+y^2-2xy\right)+\left(y^2+z^2-2yz\right)+\left(x^2+z^2-2zx\right)\right]$$

$$= \frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

Question 13:

If $x + y + z = 0$, show that $x^3+y^3+z^3-3xyz$.

Answer:

It is known that,

$$x^3+y^3+z^3-3xyz=(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$$

Put $x + y + z = 0$,

$$x^3+y^3+z^3-3xyz=(0)\left(x^2+y^2+z^2-xy-yz-zx\right)$$

$$x^3+y^3+z^3-3xyz=0$$

$$x^3+y^3+z^3=3xyz$$

Question 14:

Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Answer:

(i) $(-12)^3+(7)^3+(5)^3$

Let $x = -12$, $y = 7$, and $z = 5$

It can be observed that,

$x + y + z = -12 + 7 + 5 = 0$

It is known that if $x + y + z = 0$, then

$$x^3+y^3+z^3=3xyz$$

$\therefore (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$

= −1260

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let x = 28, y = −15, and z = −13

It can be observed that,

$x + y + z = 28 + (-15) + (-13) = 28 - 28 = 0$

It is known that if $x + y + z = 0$, then

$x^3 + y^3 + z^3 = 3xyz$

$$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$$
$$= 16380$$

Question 15:

Give possible expressions for the length and breadth of each of thefollowing rectangles, in which their areas are given:

Area: $25a^2 - 35a + 12$	Area: $35y^2 + 13y - 12$
I	II

Answer:

Area = Length × Breadth

The expression given for the area of the rectangle has to be factorised. One of its factors will be its length and the other will be its breadth.

(i) $25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12$

$= 5a(5a-3) - 4(5a-3)$

$= (5a-3)(5a-4)$

Therefore, possible length = 5a − 3

And, possible breadth = 5a − 4

(ii) $35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$= 7y(5y+4) - 3(5y+4)$

$= (5y+4)(7y-3)$

Therefore, possible length = $5y + 4$

And, possible breadth = $7y - 3$

Question 16:

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

I	II
Volume: $3x^2 - 12x$	Volume: $12ky^2 + 8ky - 20k$

Answer:

Volume of cuboid = Length × Breadth × Height

The expression given for the volume of the cuboid has to be factorised. One of its factors will be its length, one will be its breadth, and one will be its height.

(i) $3x^2 - 12x = 3x(x-4)$

One of the possible solutions is as follows.

Length = 3, Breadth = x, Height = $x - 4$

(ii) $12ky^2 + 8ky - 20k = 4k(3y^2 + 2y - 5)$

$= 4k[3y^2 + 5y - 3y - 5]$

$= 4k[y(3y+5) - 1(3y+5)]$

$= 4k(3y+5)(y-1)$

One of the possible solutions is as follows.

Length = $4k$, Breadth = $3y + 5$, Height = $y - 1$

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Class 9th Maths
Chapter 3: Coordinate Geometry

Exercise 2.1

Question 1:

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$ (v) $x^{10} + y^3 + t^{50}$

Answer:

(i) $4x^2 - 3x + 7$

Yes, this expression is a polynomial in one variable x.

(ii) $y^2 + \sqrt{2}$

Yes, this expression is a polynomial in one variable y.

(iii) $3\sqrt{t} + t\sqrt{2}$

No. It can be observed that the exponent of variable t in term $3\sqrt{t}$ is $\frac{1}{2}$, which is not a whole number. Therefore, this expression is not a polynomial.

(iv) $y + \frac{2}{y}$

No. It can be observed that the exponent of variable y in term $\frac{2}{y}$ is -1, which is not a whole number. Therefore, this expression is not a polynomial.

(v) $x^{10} + y^3 + t^{50}$

No. It can be observed that this expression is a polynomial in 3 variables x, y, and t. Therefore, it is not a polynomial in one variable.

Question 2:

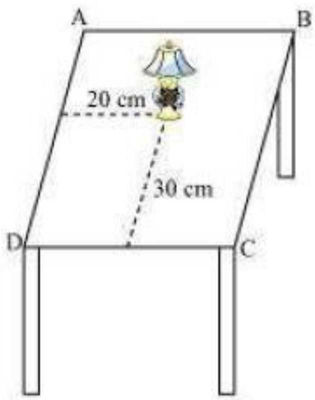
Write the coefficients of x^2 in each of the following:

Exercise 3.1

Question 1:

How will you describe the position of a table lamp on your study table to another person?

Answer:



Consider that the lamp is placed on the table. Choose two adjacent edges, DC and AD. Then, draw perpendiculars on the edges DC and AD from the position of lamp and measure the lengths of these perpendiculars. Let the length of these perpendiculars be 30 cm and 20 cm respectively. Now, the position of the lamp from the left edge (AD) is 20 cm and from the lower edge (DC) is 30 cm. This can also be written as (20, 30), where 20 represents the perpendicular distance of the lamp from edge AD and 30 represents the perpendicular distance of the lamp from edge DC.

Question 2:

(Street Plan): A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West direction.

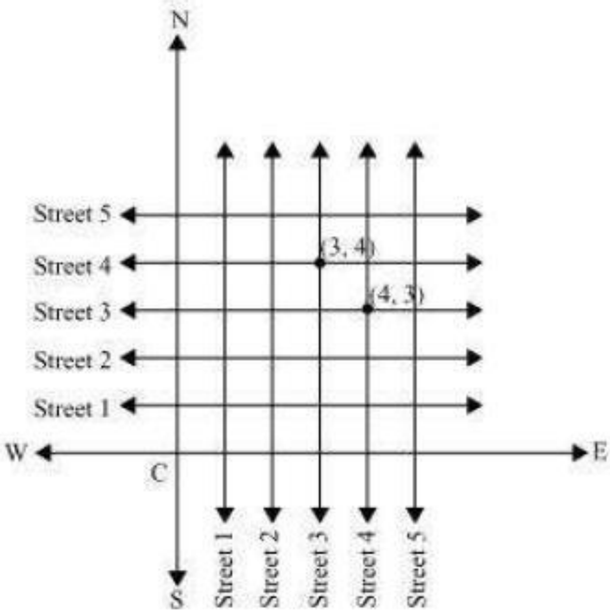
All the other streets of the city run parallel to these roads and are 200 m apart. There are about 5 streets in each direction. Using 1 cm = 100 m, draw a model of the city on your notebook Represent the roads/streets by single lines.

There are many cross-streets in your model. A particular cross-street is made by two streets, one running in the North-South direction and another in the East-West direction. Each cross street is referred to in the following manner: If the 2nd street running in the North-South direction and 5th in the East-West direction meet at some crossing, then we will call this cross-street (2, 5). Using this convention, find:

(i) How many cross - streets can be referred to as (4, 3).

(ii) How many cross - streets can be referred to as (3, 4).

Answer:



Both the cross-streets are marked in the above figure. It can be observed that there is only one cross-street which can be referred as (4, 3), and again, only one which can be referred as (3, 4).

Exercise 3.2

Question 1:

Write the answer of each of the following questions:

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

(ii) What is the name of each part of the plane formed by these two lines?

(iii) Write the name of the point where these two lines intersect.

Answer:

(i) The name of horizontal lines and vertical lines drawn to determine the position of any point in the Cartesian plane is *x*-axis and *y*-axis respectively.

(ii) The name of each part of the plane formed by these two lines, *x*-axis and *y*-axis, is quadrant (one-fourth part).

(iii) The name of the point where these two lines intersect is the origin.

Question 2:

See the given figure, and write the following:

(i) The coordinates of B.

(ii) The coordinates of C.

(iii) The point identified by the coordinates $(-3,-5)$.

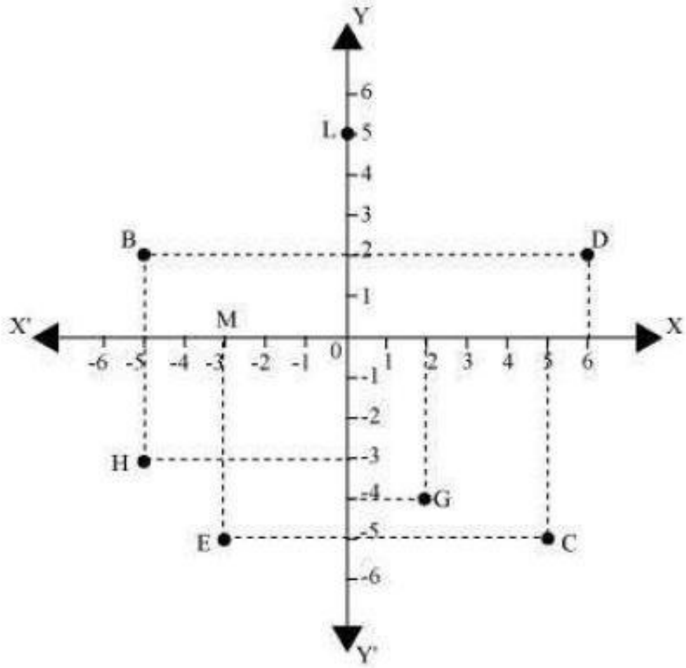
(iv) The point identified by the coordinates $(2,-4)$

(v) The abscissa of the point D.

(vi) The ordinate of the point H.

(vii) The coordinates of the point L.

(viii) The coordinates of the point M



Answer:

(i) The *x*-coordinate and the *y*-coordinate of point B are −5 and 2 respectively. Therefore, the coordinates of point B are (−5, 2).

(ii) The *x*-coordinate and the *y*-coordinate of point C are 5 and −5 respectively. Therefore, the coordinates of point C are (5, −5).

(iii) The point whose *x*-coordinate and *y*-coordinate are −3 and −5 respectively is point E.

(iv) The point whose *x*-coordinate and *y*-coordinate are 2 and −4 respectively is point G.

(v) The *x*-coordinate of point D is 6. Therefore, the abscissa of point D is 6.

(vi) The *y*-coordinate of point H is −3. Therefore, the ordinate of point H is −3.

(vii) The *x*-coordinate and the *y*-coordinate of point L are 0 and 5 respectively. Therefore, the coordinates of point L are (0, 5).

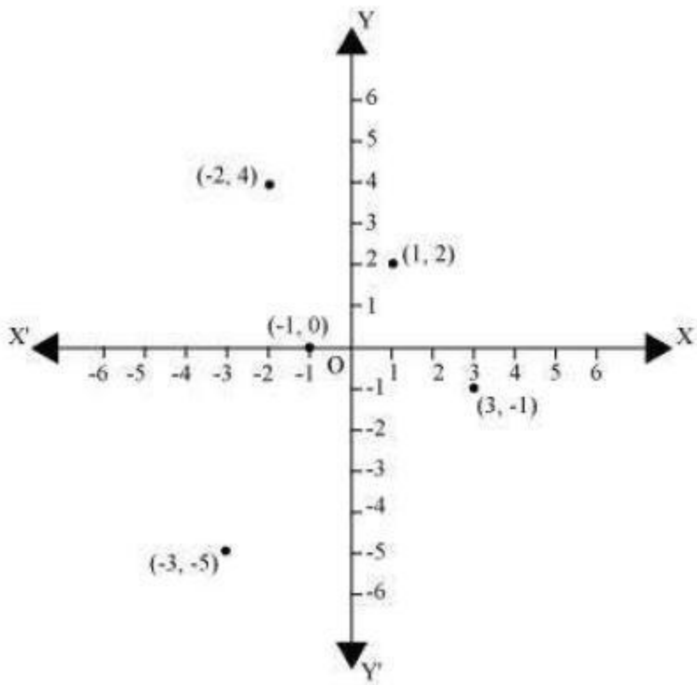
(viii) The *x*-coordinate and the *y*-coordinate of point M are −3 and 0 respectively. Therefore, the coordinates of point M is (−3, 0).

Exercise 3.3

Question 1:

In which quadrant or on which axis do each of the points $(-2,4),(3,-1),(-1,0),(1,2)$ and $(-3,-5)$ lie? Verify your answer by locating them on the Cartesian plane.

Answer:



The point $(-2,4)$ lies in the IInd quadrant in the Cartesian plane because for point $(-2,4)$, *x*-coordinate is negative and *y*-coordinate is positive.

Again, the point $(3,-1)$ lies in the IVth quadrant in the Cartesian plane because for point $(3,-1)$, *x*-coordinate is positive and *y*-coordinate is negative.

The point $(-1,0)$ lies on negative *x*-axis because for point $(-1,0)$, the value of *y*-coordinate is zero and the value of *x*-coordinate is negative.

The point $(1,2)$ lies in the I[st] quadrant as for point $(1,2)$, both *x* and *y* are positive.

The point $(-3,-5)$ lies in the III[rd] quadrant in the Cartesian plane because for point $(-3,-5)$, both *x* and *y* are negative.

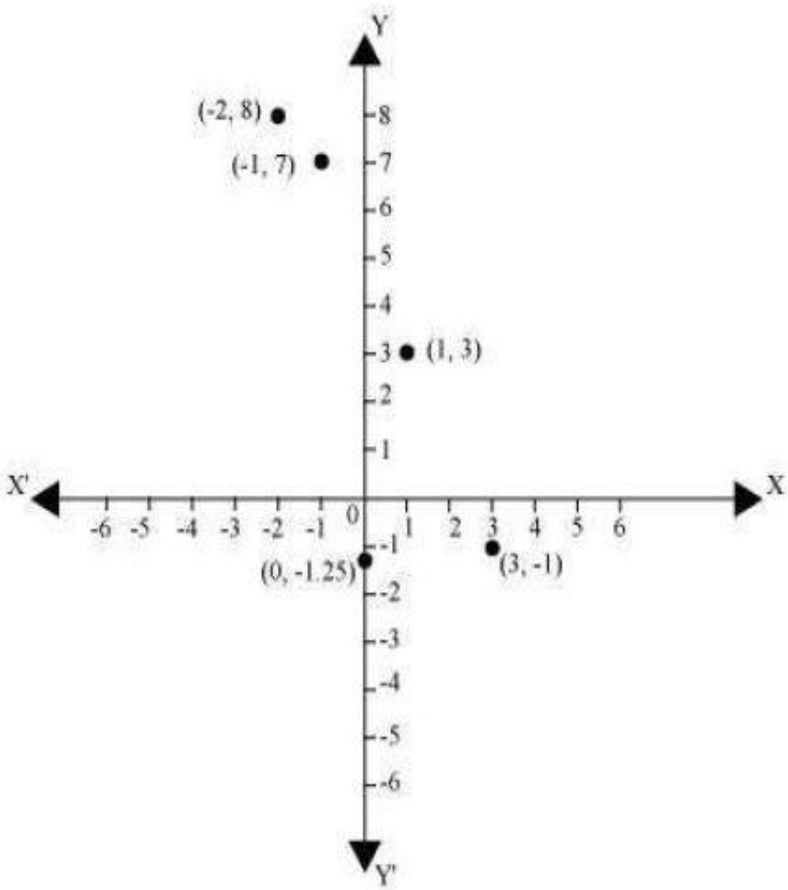
Question 2:

Plot the point (*x*, *y*) given in the following table on the plane, choosing suitable units of distance on the axis.

x	− 2	− 1	0	1	3
y	8	7	1.25	3	− 1

Answer:

The given points can be plotted on the Cartesian plane as follows.



NCERT

Class 9th Maths

Chapter 4: Linear Equations in Two Variable

Exercise 4.1

Question 1:

The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

(Take the cost of a notebook to be Rs x and that of a pen to be Rs y.)

Answer:

Let the cost of a notebook and a pen be x and y respectively.

Cost of notebook = 2 × Cost of pen

$x = 2y$

$x - 2y = 0$

Question 2:

Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b, c in each case:

(i) $2x + 3y = 9.3\overline{5}$ (ii) $x - \frac{y}{5} - 10 = 0$ (iii) $-2x + 3y = 6$

(iv) $x = 3y$ (v) $2x = -5y$ (vi) $3x + 2 = 0$

(vii) $y - 2 = 0$ (viii) $5 = 2x$

Answer:

(i) $2x + 3y = 9.3\overline{5}$

$2x + 3y - 9.3\overline{5} = 0$

Comparing this equation with $ax + by + c = 0$,

$a = 2$, $b = 3$, $c = -9.3\overline{5}$

(ii) $x - \frac{y}{5} - 10 = 0$

Comparing this equation with $ax + by + c = 0$,

$a = 1$, $b = -\frac{1}{5}$, $c = -10$

(iii) $-2x + 3y = 6$

$-2x + 3y - 6 = 0$

Comparing this equation with $ax + by + c = 0$,

$a = -2, b = 3, c = -6$

(iv) $x = 3y$

$1x - 3y + 0 = 0$

Comparing this equation with $ax + by + c = 0$,

$a = 1, b = -3, c = 0$

(v) $2x = -5y$

$2x + 5y + 0 = 0$

Comparing this equation with $ax + by + c = 0$,

$a = 2, b = 5, c = 0$

(vi) $3x + 2 = 0$

$3x + 0.y + 2 = 0$

Comparing this equation with $ax + by + c = 0$,

$a = 3, b = 0, c = 2$

(vii) $y - 2 = 0$

$0.x + 1.y - 2 = 0$

Comparing this equation with $ax + by + c = 0$,

$a = 0, b = 1, c = -2$

(vii) $5 = 2x$

$-2x + 0.y + 5 = 0$

Comparing this equation with $ax + by + c = 0$,

$a = -2, b = 0, c = 5$

Exercise 4.2

Question 1:

Which one of the following options is true, and why?

$y = 3x + 5$ has

(i) a unique solution, (ii) only two solutions, (iii) infinitely many solutions

Answer:

$y = 3x + 5$ is a linear equation in two variables and it has infinite possible solutions. As for every value of x, there will be a value of y satisfying the above equation and vice-versa.

Hence, the correct answer is (iii).

Question 2:

Write four solutions for each of the following equations:

(i) $2x + y = 7$ (ii) $\pi x + y = 9$ (iii) $x = 4y$

Answer:

(i) $2x + y = 7$

For $x = 0$,

$2(0) + y = 7$

$\Rightarrow y = 7$

Therefore, (0, 7) is a solution of this equation.

For $x = 1$,

$2(1) + y = 7$

$\Rightarrow y = 5$

Therefore, (1, 5) is a solution of this equation.

For $x = -1$,

$2(-1) + y = 7$

$\Rightarrow y = 9$

Therefore, (−1, 9) is a solution of this equation.

For $x = 2$,

$2(2) + y = 7$

$\Rightarrow y = 3$

Therefore, (2, 3) is a solution of this equation.

(ii) $\pi x + y = 9$

For $x = 0$,

$\pi(0) + y = 9$

$\Rightarrow y = 9$

Therefore, (0, 9) is a solution of this equation.

For $x = 1$,

$\pi(1) + y = 9$

$\Rightarrow y = 9 - \pi$

Therefore, $(1, 9 - \pi)$ is a solution of this equation.

For $x = 2$,

$\pi(2) + y = 9$

$\Rightarrow y = 9 - 2\pi$

Therefore, $(2, 9 - 2\pi)$ is a solution of this equation.

For $x = -1$,

$\pi(-1) + y = 9$

$\Rightarrow y = 9 + \pi$

$\Rightarrow (-1, 9 + \pi)$ is a solution of this equation.

(iii) $x = 4y$

For $x = 0$,

$0 = 4y$

$\Rightarrow y = 0$

Therefore, (0, 0) is a solution of this equation.

For $y = 1$,

$x = 4(1) = 4$

Therefore, (4, 1) is a solution of this equation.

For $y = -1$,

$x = 4(-1)$

$\Rightarrow x = -4$

Therefore, $(-4, -1)$ is a solution of this equation.

For $x = 2$,

$2 = 4y$

$\Rightarrow y = \frac{2}{4} = \frac{1}{2}$

Therefore, $\left(2, \frac{1}{2}\right)$ is a solution of this equation.

Question 3:

Check which of the following are solutions of the equation $x - 2y = 4$ and which are not:

(i) (0, 2 (ii) (2, 0) (iii) (4, 0)

(iv) $\left(\sqrt{2}, 4\sqrt{2}\right)$ (v) (1, 1)

Answer:

(i) (0, 2)

Putting $x = 0$ and $y = 2$ in the L.H.S of the given equation,

$x - 2y = 0 - 2\times2 = -4 \neq 4$

L.H.S ≠ R.H.S

Therefore, (0, 2) is not a solution of this equation.

(ii) (2, 0)

Putting $x = 2$ and $y = 0$ in the L.H.S of the given equation,

$x - 2y = 2 - 2 \times 0 = 2 \neq 4$

L.H.S ≠ R.H.S

Therefore, (2, 0) is not a solution of this equation.

(iii) (4, 0)

Putting $x = 4$ and $y = 0$ in the L.H.S of the given equation,

$x - 2y = 4 - 2(0)$

$= 4 =$ R.H.S

Therefore, (4, 0) is a solution of this equation.

(iv) $\left(\sqrt{2}, 4\sqrt{2}\right)$

Putting $x=\sqrt{2}$ and $y=4\sqrt{2}$ in the L.H.S of the given equation,

$$x-2y=\sqrt{2}-2\left(4\sqrt{2}\right)$$
$$=\sqrt{2}-8\sqrt{2}=-7\sqrt{2}\neq 4$$

L.H.S ≠ R.H.S

Therefore, $\left(\sqrt{2},\,4\sqrt{2}\right)$ is not a solution of this equation.

(v) (1, 1)

Putting x = 1 and y = 1 in the L.H.S of the given equation,

$x - 2y = 1 - 2(1) = 1 - 2 = -1 \neq 4$

L.H.S ≠ R.H.S

Therefore, (1, 1) is not a solution of this equation.

Question 4:

Find the value of k, if x = 2, y = 1 is a solution of the equation $2x + 3y = k$.

Answer:

Putting x = 2 and y = 1 in the given equation,

$2x + 3y = k$

$\Rightarrow 2(2) + 3(1) = k$

$\Rightarrow 4 + 3 = k$

$\Rightarrow k = 7$

Therefore, the value of k is 7.

Exercise 4.3

Question 1:

Draw the graph of each of the following linear equations in two variables:

(i) $x+y=4$ (ii) $x-y=2$ (iii) $y = 3x$ (iv) $3 = 2x + y$

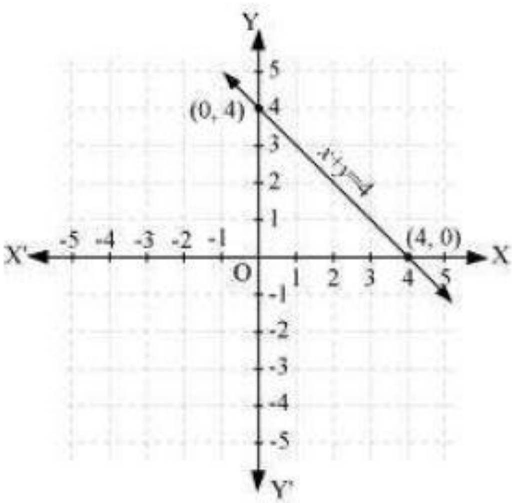
Answer:

(i) $x+y=4$

It can be observed that $x = 0$, $y = 4$ and $x = 4$, $y = 0$ are solutions of the above equation. Therefore, the solution table is as follows.

x	0	4
y	4	0

The graph of this equation is constructed as follows.

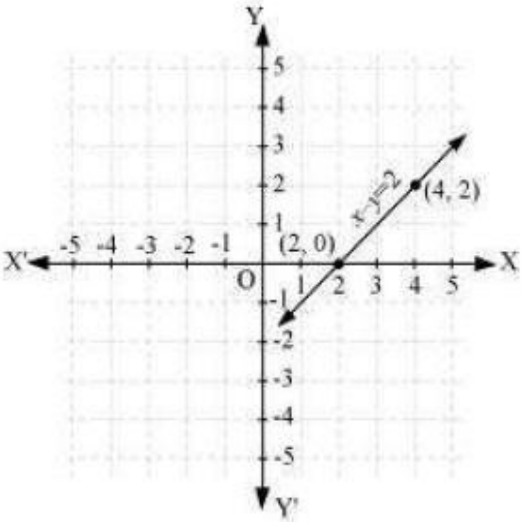


(ii) $x-y=2$

It can be observed that $x = 4$, $y = 2$ and $x = 2$, $y = 0$ are solutions of the above equation. Therefore, the solution table is as follows.

x	4	2
y	2	0

The graph of the above equation is constructed as follows.

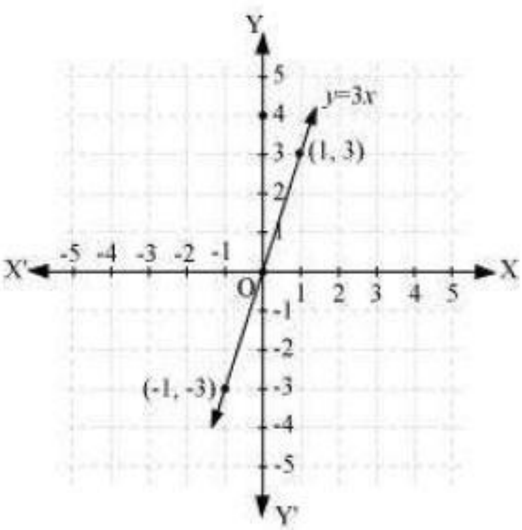


(iii) $y = 3x$

It can be observed that $x = -1$, $y = -3$ and $x = 1$, $y = 3$ are solutions of the above equation. Therefore, the solution table is as follows.

x	− 1	1
y	− 3	3

The graph of the above equation is constructed as follows.

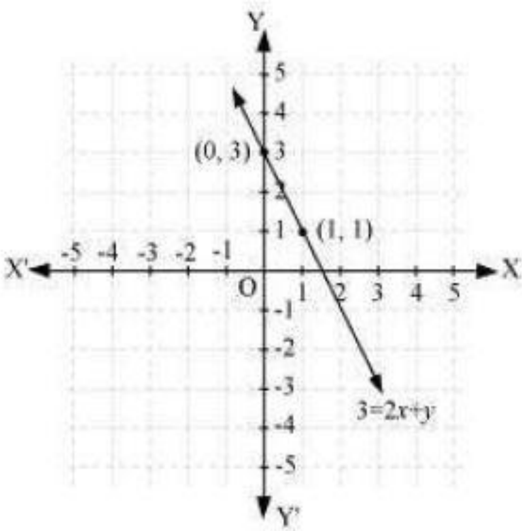


(iv) $3 = 2x + y$

It can be observed that $x = 0$, $y = 3$ and $x = 1$, $y = 1$ are solutions of the above equation. Therefore, the solution table is as follows.

x	0	1
y	3	1

The graph of this equation is constructed as follows.



Question 2:

Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?

Answer:

It can be observed that point (2, 14) satisfies the equation $7x - y = 0$ and $x - y + 12 = 0$.

Therefore, $7x - y = 0$ and $x - y + 12 = 0$ are two lines passing through point (2, 14).

As it is known that through one point, infinite number of lines can pass through, therefore, there are infinite lines of such type passing through the given point.

Question 3:

If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.

Answer:

Putting $x = 3$ and $y = 4$ in the given equation,

$3y = ax + 7$

$3\ (4) = a\ (3) + 7$

$5 = 3a$

$a = \frac{5}{3}$

Question 4:

The taxi fare in a city is as follows: For the first kilometre, the fares is Rs 8 and for the subsequent distance it is Rs 5 per km. Taking the distance covered as x km and total fare as Rs y, write a linear equation for this information, and draw its graph.

Answer:

Total distance covered = x km

Fare for 1st kilometre = Rs 8

Fare for the rest of the distance = Rs $(x - 1)\ 5$

Total fare = Rs $[8 + (x - 1)\ 5]$

$y = 8 + 5x - 5$

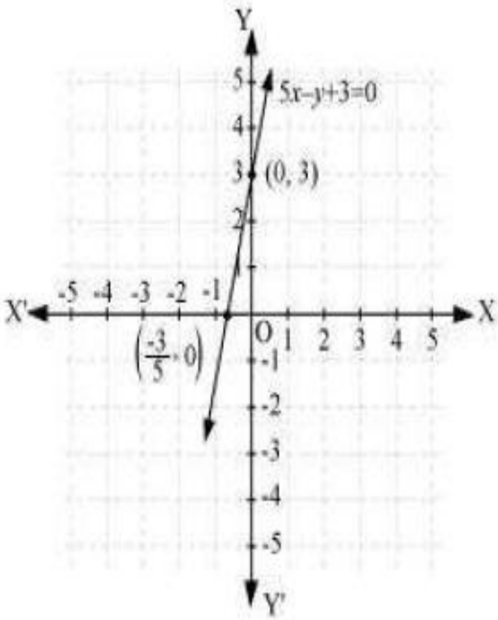
$y = 5x + 3$

$5x - y + 3 = 0$

It can be observed that point (0, 3) and $\left(-\frac{3}{5}, 0\right)$ satisfies the above equation. Therefore, these are the solutions of this equation.

x	0	$\frac{-3}{5}$
y	3	0

The graph of this equation is constructed as follows.

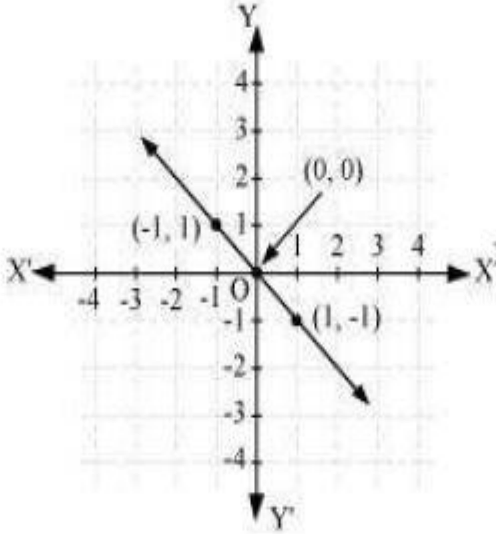


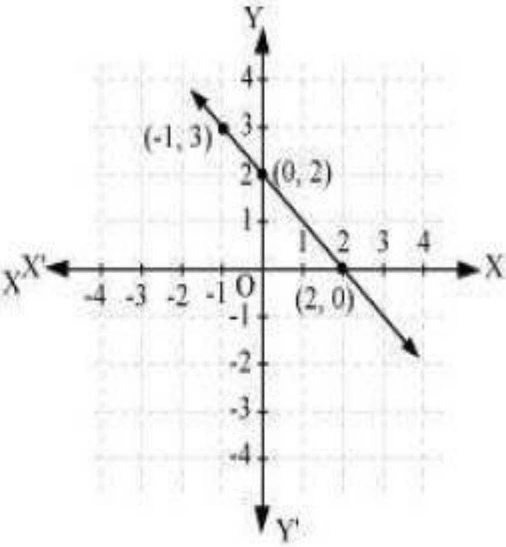
Here, it can be seen that variable x and y are representing the distance covered and the fare paid for that distance respectively and these quantities may not be negative. Hence, only those values of x and y which are lying in the 1st quadrant will be considered.

Question 5:

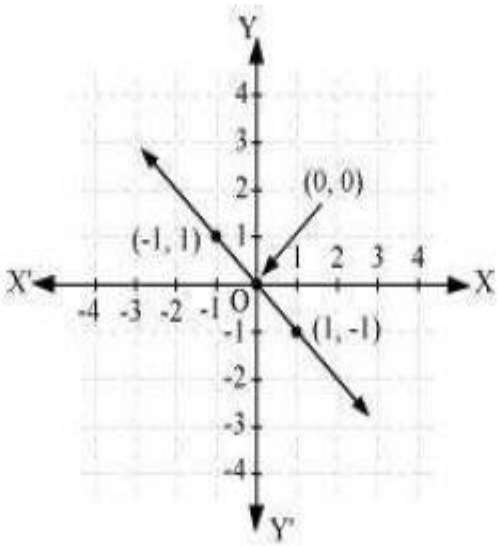
From the choices given below, choose the equation whose graphs are given in the given figures.

For the first figure	For the second figure
(i) $y = x$	(i) $y = x + 2$
(ii) $x + y = 0$	(ii) $y = x - 2$
(iii) $y = 2x$	(iii) $y = -x + 2$
(iv) $2 + 3y = 7x$	(iv) $x + 2y = 6$





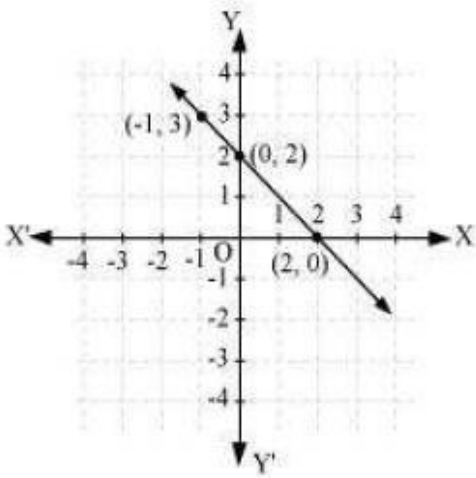
Answer:



Points on the given line are (−1, 1), (0, 0), and (1, −1).

It can be observed that the coordinates of the points of the graph satisfy the equation $x + y = 0$. Therefore, $x + y = 0$ is the equation corresponding to the graph as shown in the first figure.

Hence, (ii) is the correct answer.



Points on the given line are (−1, 3), (0, 2), and (2, 0). It can be observed that the coordinates of the points of the graph satisfy the equation $y = -x + 2$.

Therefore, $y = -x + 2$ is the equation corresponding to the graph shown in the second figure.

Hence, (iii) is the correct answer.

Question 6:

If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is

(i) 2 units (ii) 0 units

Answer:

Let the distance travelled and the work done by the body be x and y respectively.

Work done $\propto$ distance travelled

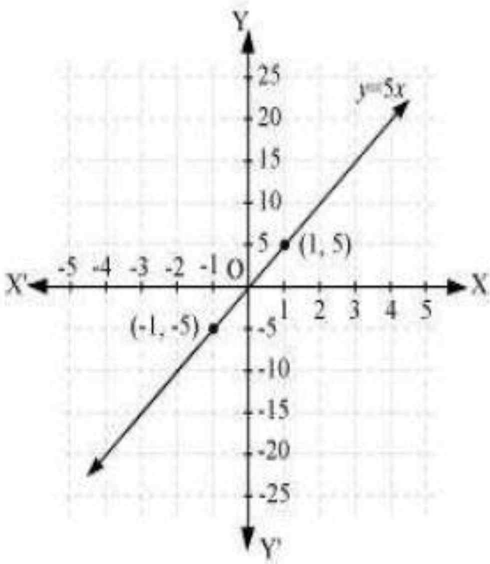
$y \propto x$

$y = kx$

Where, k is a constant

If constant force is 5 units, then work done $y = 5x$

It can be observed that point (1, 5) and (−1, −5) satisfy the above equation. Therefore, these are the solutions of this equation. The graph of this equation is constructed as follows.



(i)From the graphs, it can be observed that the value of y corresponding to x = 2 is 10. This implies that the work done by the body is 10 units when the distance travelled by it is 2 units.

(ii) From the graphs, it can be observed that the value of y corresponding to x = 0 is 0. This implies that the work done by the body is 0 units when the distance travelled by it is 0 unit.

Question 7:

Yamini and Fatima, two students of Class IX of a school, together contributed Rs 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as Rs x and Rs y.) Draw the graph of the same.

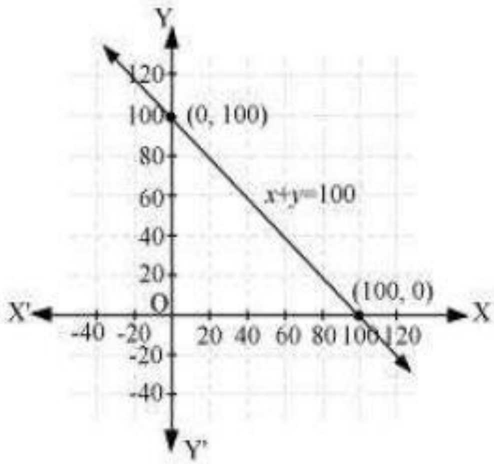
Answer:

Let the amount that Yamini and Fatima contributed be x and y respectively towards the Prime Minister's Relief fund.

Amount contributed by Yamini + Amount contributed by Fatima = 100

$x + y = 100$

It can be observed that (100, 0) and (0, 100) satisfy the above equation. Therefore, these are the solutions of the above equation. The graph is constructed as follows.



Here, it can be seen that variable x and y are representing the amount contributed by Yamini and Fatima respectively and these quantities cannot be negative. Hence, only those values of x and y which are lying in the 1^{st} quadrant will be considered.

Question 8:

In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95°F, what is the temperature in Celsius?

(iv) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

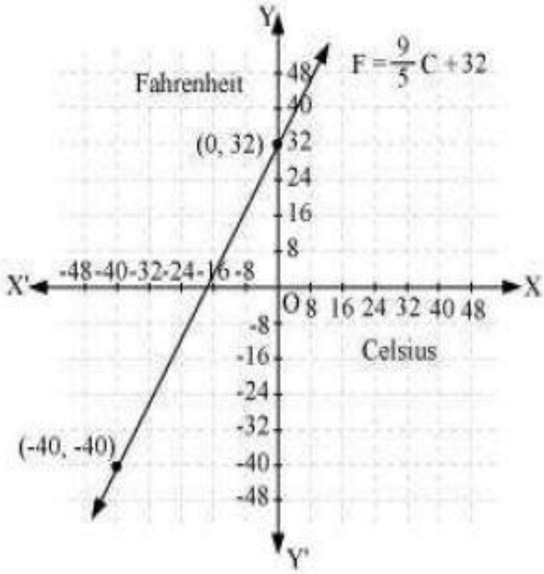
(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

Answer:

(i) $$F = \left(\frac{9}{5}\right)C + 32$$

It can be observed that points (0, 32) and (−40, −40) satisfy the given equation. Therefore, these points are the solutions of this equation.

The graph of the above equation is constructed as follows.



(ii) Temperature = 30°C

$$F = \left(\frac{9}{5}\right)C + 32$$

$$F = \left(\frac{9}{5}\right)30 + 32 = 54 + 32 = 86$$

Therefore, the temperature in Fahrenheit is 86°F.

(iii) Temperature = 95°F

$$F = \left(\frac{9}{5}\right)C + 32$$

$$95 = \left(\frac{9}{5}\right)C + 32$$

$$63 = \left(\frac{9}{5}\right)C$$

$$C = 35$$

Therefore, the temperature in Celsius is 35°C.

(iv) $F = \left(\frac{9}{5}\right)C + 32$

If C = 0°C, then

$$F = \left(\frac{9}{5}\right)0 + 32 = 32$$

Therefore, if C = 0°C, then F = 32°F

If F = 0°F, then

$$0 = \left(\frac{9}{5}\right)C + 32$$

$$\left(\frac{9}{5}\right)C = -32$$

$$C = \frac{-160}{9} = -17.77$$

Therefore, if F = 0°F, then C = −17.8°C

(v) $F = \left(\frac{9}{5}\right)C + 32$

Here, F = C

$$F = \left(\frac{9}{5}\right)F + 32$$

$$\left(\frac{9}{5} - 1\right)F + 32 = 0$$

$$\left(\frac{4}{5}\right)F = -32$$

$$F = -40$$

Yes, there is a temperature, −40°, which is numerically the same in both Fahrenheit and Celsius.

Exercise 4.4

Question 1:

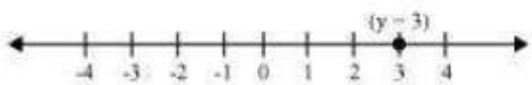
Give the geometric representation of $y = 3$ as an equation

(I) in one variable

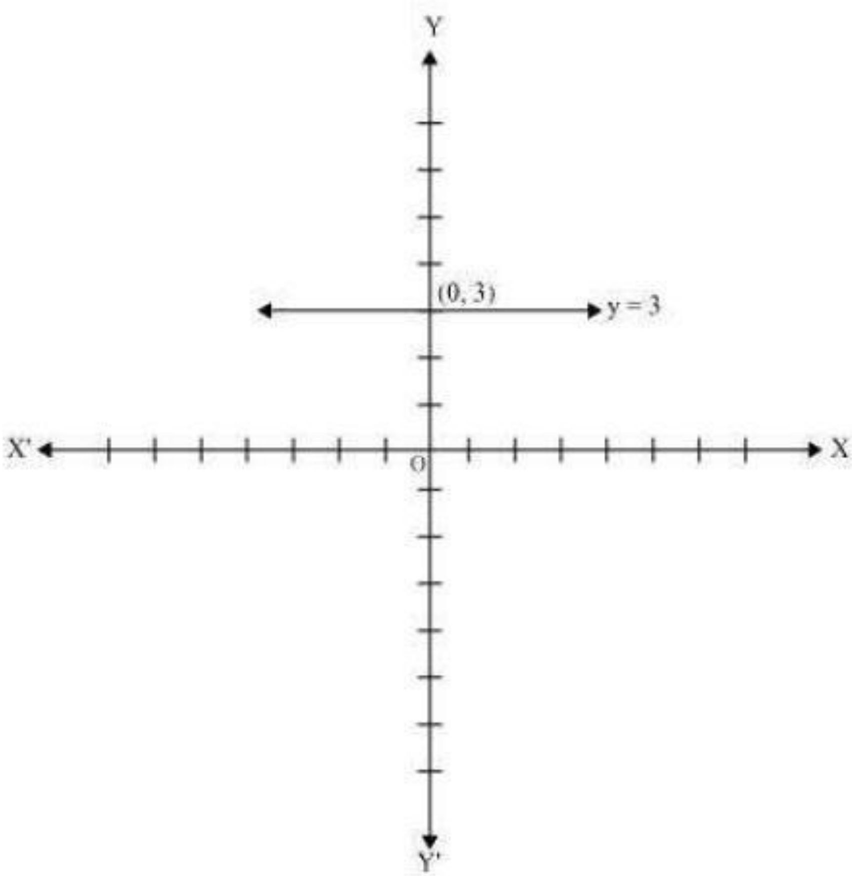
(II) in two variables

Answer:

In one variable, $y = 3$ represents a point as shown in following figure.



In two variables, $y = 3$ represents a straight line passing through point (0, 3) and parallel to x-axis. It is a collection of all points of the plane, having their y-coordinate as 3.



Question 2:

Give the geometric representations of $2x + 9 = 0$ as an equation

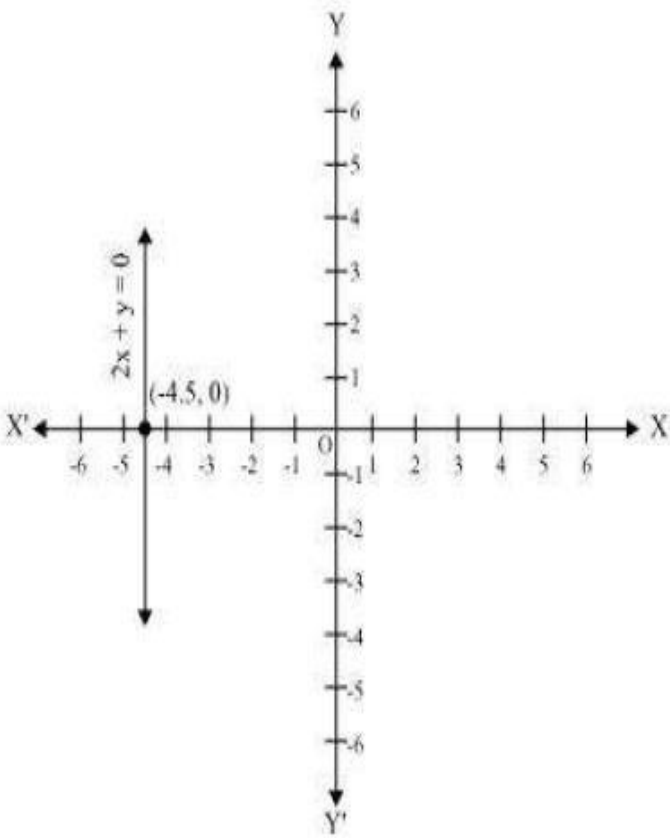
(1) in one variable

(2) in two variables

Answer:

(1) In one variable, $2x + 9 = 0$ represents a point $x = \frac{-9}{2} = -4.5$ as shown in the following figure.

(2) In two variables, $2x + 9 = 0$ represents a straight line passing through point $(-4.5, 0)$ and parallel to y-axis. It is a collection of all points of the plane, having their x-coordinate as 4.5.

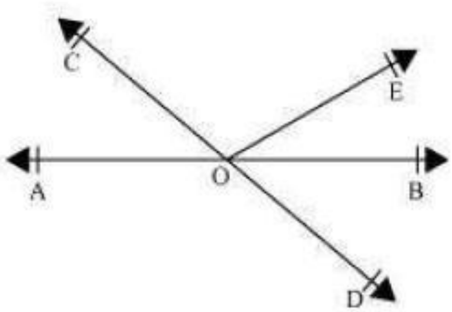


NCERT
Class 9th Maths
Chapter 6: Lines and Angles

Exercise 6.1

Question 1:

In the given figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle$BOE and reflex $\angle$COE.



Answer:

AB is a straight line, rays OC and OE stand on it.

$\therefore \angle AOC + \angle COE + \angle BOE = 180°$

$\Rightarrow (\angle AOC + \angle BOE) + \angle COE = 180°$

$\Rightarrow 70° + \angle COE = 180°$

$\Rightarrow \angle COE = 180° - 70° = 110°$

Reflex $\angle COE = 360° - 110° = 250°$

CD is a straight line, rays OE and OB stand on it.

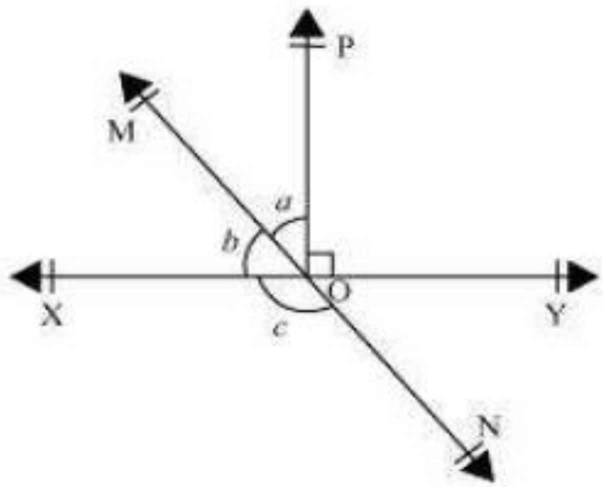
$\therefore \angle COE + \angle BOE + \angle BOD = 180°$

$\Rightarrow 110° + \angle BOE + 40° = 180°$

$\Rightarrow \angle BOE = 180° - 150° = 30°$

Question 2:

In the given figure, lines XY and MN intersect at O. If $\angle$POY = $90°$ and a:b = 2 : 3, find c.



Answer:

Let the common ratio between a and b be x.

$\therefore a = 2x$, and $b = 3x$

XY is a straight line, rays OM and OP stand on it.

$\therefore \angle XOM + \angle MOP + \angle POY = 180^\circ$

$b + a + \angle POY = 180^\circ$

$3x + 2x + 90^\circ = 180^\circ$

$5x = 90^\circ$

$x = 18^\circ$

$a = 2x = 2 \times 18 = 36^\circ$

$b = 3x = 3 \times 18 = 54^\circ$

MN is a straight line. Ray OX stands on it.

$\therefore b + c = 180^\circ$ (Linear Pair)

$54^\circ + c = 180^\circ$

$c = 180^\circ - 54^\circ = 126^\circ$

$\therefore c = 126^\circ$

Question 3:

It can be observed that,

$x + y + z + w = 360^\circ$ (Complete angle)

It is given that,

$x + y = z + w$

$\therefore x + y + x + y = 360^\circ$

$2(x + y) = 360^\circ$

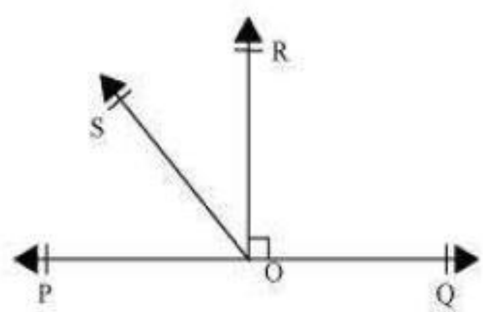
$x + y = 180^\circ$

Since x and y form a linear pair, AOB is a line.

Question 5:

In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS).$$



Answer:

It is given that OR ⊥ PQ

□ □POR = 90º

□ □POS + □SOR = 90º

□ROS = 90º – □POS … (1)

□QOR = 90º (As OR □ PQ)

□QOS – □ROS = 90º

□ROS = □QOS – 90º … (2)

On adding equations (1) and (2), we obtain

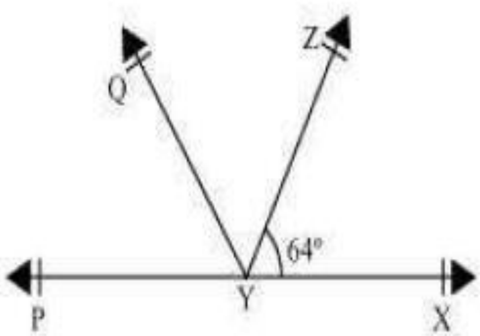
2 □ROS = □QOS – □POS

□ROS = $\frac{1}{2}$ (□QOS – □POS)

Question 6:

It is given that □XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects □ZYP, find □XYQ and reflex □QYP.

Answer:



It is given that line YQ bisects □PYZ.

Hence, □QYP = □ZYQ

It can be observed that PX is a line. Rays YQ and YZ stand on it.

□ □XYZ + □ZYQ + □QYP = 180°

□ 64° + 2□QYP = 180°

□ 2□QYP = 180° – 64° = 116°

□ □QYP = 58°

Also, □ZYQ = □QYP = 58°

Reflex □QYP = 360° – 58° = 302°

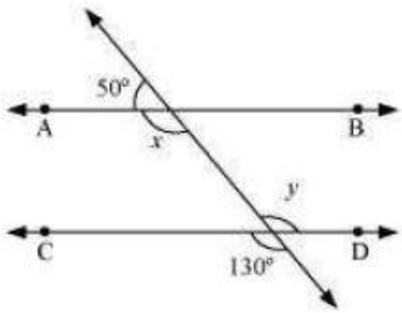
□XYQ = □XYZ + □ZYQ

= 64° + 58° = 122°

Exercise 6.2

Question 1:

In the given figure, find the values of x and y and then show that AB || CD.



Answer:

It can be observed that,

$50^\circ + x = 180^\circ$ (Linear pair)

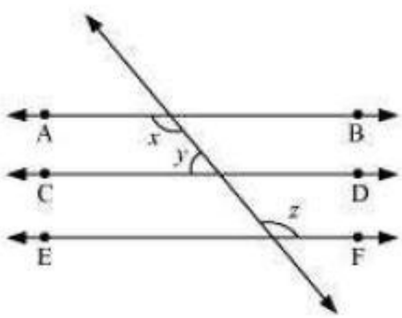
$x = 130^\circ$ ... (1)

Also, $y = 130^\circ$ (Vertically opposite angles)

As x and y are alternate interior angles for lines AB and CD and also measures of these angles are equal to each other, therefore, line AB || CD.

Question 2:

In the given figure, if AB || CD, CD || EF and y: z = 3: 7, find x.



Answer:

It is given that AB || CD and CD || EF

☐ AB || CD || EF (Lines parallel to the same line are parallel to each other)

It can be observed that

$x = z$ (Alternate interior angles) ... (1)

It is given that $y: z = 3: 7$

Let the common ratio between y and z be a.

□ $y = 3a$ and $z = 7a$

Also, $x + y = 180^o$ (Co-interior angles on the same side of the transversal)

$z + y = 180^o$ [Using equation (1)]

$7a + 3a = 180^o$

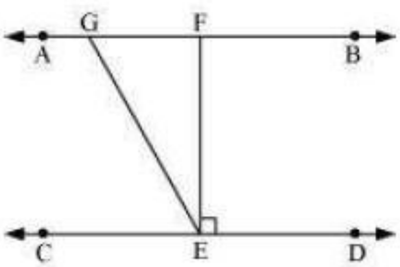
$10a = 180^o$

$a = 18^o$

□ $x = 7a = 7 \times 18^o = 126^o$

Question 3:

In the given figure, If AB || CD, EF □ CD and □GED = 126º, find □AGE, □GEF and □FGE.



Answer:

It is given that,

AB || CD

EF □ CD

□GED = 126º

□ □GEF + □FED = 126º

□ □GEF + 90º = 126º

□ □GEF = 36º

□AGE and □GED are alternate interior angles.

□ □AGE = □GED = 126º

However, □AGE + □FGE = 180º (Linear pair)

□ 126º + □FGE = 180º

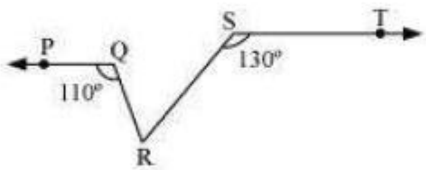
□ □FGE = 180º – 126º = 54º

□ □AGE = 126º, □GEF = 36º, □FGE = 54º

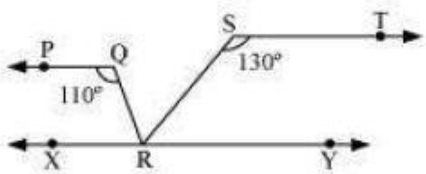
Question 4:

In the given figure, if PQ || ST, □PQR = 110º and □RST = 130º, find □QRS.

[**Hint**: Draw a line parallel to ST through point R.]



Answer:



Let us draw a line XY parallel to ST and passing through point R.

□PQR + □QRX = 180º (Co-interior angles on the same side of transversal QR)

□ 110º + □QRX = 180º

□ □QRX = 70º

Also,

□RST + □SRY = 180º (Co-interior angles on the same side of transversal SR)

130º + □SRY = 180º

□SRY = 50º

XY is a straight line. RQ and RS stand on it.

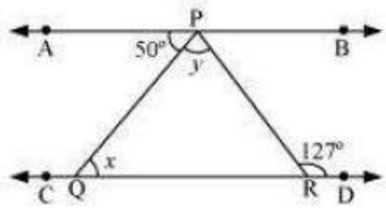
□ □QRX + □QRS + □SRY = 180º

70º + □QRS + 50º = 180º

□QRS = 180º – 120º = 60º

Question 5:

In the given figure, if AB || CD, □APQ = 50º and □PRD = 127º, find *x* and *y*.



Answer:

□APR = □PRD (Alternate interior angles)

$50^o + y = 127^o$

$y = 127^o - 50^o$

$y = 77^o$

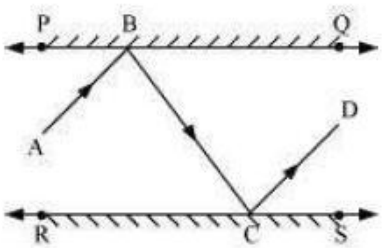
Also, □APQ = □PQR (Alternate interior angles)

$50^o = x$

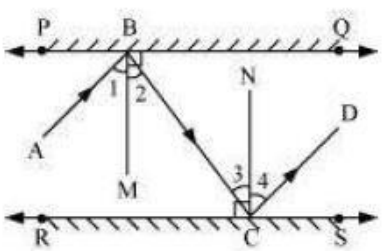
□ $x = 50^o$ and $y = 77^o$

Question 6:

In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



Answer:



Let us draw BM □ PQ and CN □ RS.

As PQ || RS

Therefore, BM || CN

Thus, BM and CN are two parallel lines and a transversal line BC cuts them at B and C respectively.

□□2 = □3 (Alternate interior angles)

However, □1 = □2 and □3 = □4 (By laws of reflection)

□ □1 = □2 = □3 = □4

Also, □1 + □2 = □3 + □4

□ABC = □DCB

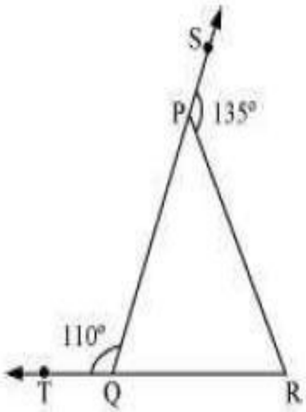
However, these are alternate interior angles.

□ AB || CD

Exercise 6.3

Question 1:

In the given figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If ☐SPR = 135º and ☐PQT = 110º, find ☐PRQ.



Answer:

It is given that,

☐SPR = 135º and ☐PQT = 110º

☐SPR + ☐QPR = 180º (Linear pair angles)

☐ 135º + ☐QPR = 180º

☐ ☐QPR = 45º

Also, ☐PQT + ☐PQR = 180º (Linear pair angles)

☐ 110º + ☐PQR = 180º

☐ ☐PQR = 70º

As the sum of all interior angles of a triangle is 180º, therefore, for ΔPQR,

☐QPR + ☐PQR + ☐PRQ = 180º

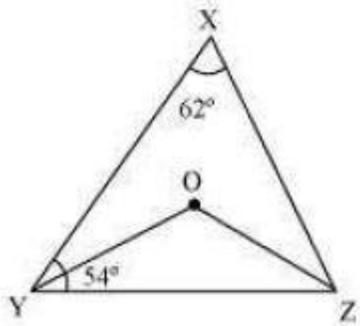
☐ 45º + 70º + ☐PRQ = 180º

☐ ☐PRQ = 180º − 115º

☐ ☐PRQ = 65º

Question 2:

In the given figure, ☐X = 62º, ☐XYZ = 54º. If YO and ZO are the bisectors of ☐XYZ and ☐XZY respectively of ΔXYZ, find ☐OZY and ☐YOZ.



Answer:

As the sum of all interior angles of a triangle is 180º, therefore, for ΔXYZ,

□X + □XYZ + □XZY = 180º

62º + 54º + □XZY = 180º

□XZY = 180º – 116º

□XZY = 64º

□OZY = $\frac{64}{2}$ = 32º (OZ is the angle bisector of □XZY)

Similarly, □OYZ = $\frac{54}{2}$ = 27º

Using angle sum property for ΔOYZ, we obtain

□OYZ + □YOZ + □OZY = 180º

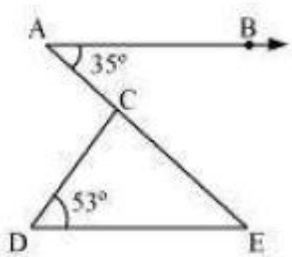
27º + □YOZ + 32º = 180º

□YOZ = 180º – 59º

□YOZ = 121º

Question 3:

In the given figure, if AB || DE, □BAC = 35º and □CDE = 53º, find □DCE.



Answer:

AB || DE and AE is a transversal.

□BAC = □CED (Alternate interior angles)

□ □CED = 35°

In ΔCDE,

□CDE + □CED + □DCE = 180° (Angle sum property of a triangle)

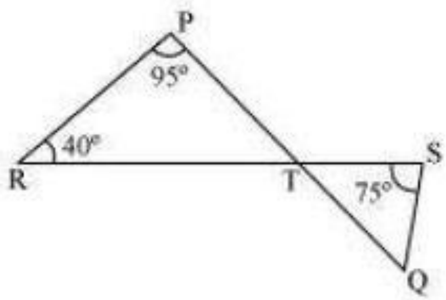
53° + 35° + □DCE = 180°

□DCE = 180° – 88°

□DCE = 92°

Question 4:

In the given figure, if lines PQ and RS intersect at point T, such that □PRT = 40 □RPT = 95° and □TSQ = 75°, find □SQT.



Answer:

Using angle sum property for ΔPRT, we obtain

□PRT + □RPT + □PTR = 180°

40° + 95° + □PTR = 180°

□PTR = 180° – 135°

□PTR = 45°

□STQ = □PTR = 45° (Vertically opposite angles)

□STQ = 45°

By using angle sum property for ΔSTQ, we obtain

□STQ + □SQT + □QST = 180°

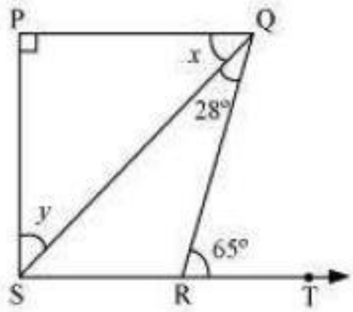
45° + □SQT + 75° = 180°

□SQT = 180° – 120°

□SQT = 60°

Question 5:

In the given figure, if PQ □ PS, PQ || SR, □SQR = 28º and □QRT = 65º, then find the values of x and y.



Answer:

It is given that PQ || SR and QR is a transversal line.

□PQR = □QRT (Alternate interior angles)

$x + 28º = 65º$

$x = 65º - 28º$

$x = 37º$

By using the angle sum property for ΔSPQ, we obtain

□SPQ + $x + y = 180º$

$90º + 37º + y = 180º$

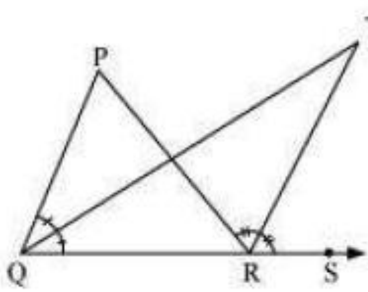
$y = 180º - 127º$

$y = 53º$

$\therefore x = 37º$ and $y = 53º$

Question 6:

In the given figure, the side QR of ΔPQR is produced to a point S. If the bisectors of □PQR and □PRS meet at point T, then prove that □QTR= $\frac{1}{2}$ □QPR.



Answer:

In ΔQTR, □TRS is an exterior angle.

∴ □QTR + □TQR = □TRS

□QTR = □TRS − □TQR (1)

For ΔPQR, □PRS is an external angle.

∴ □QPR + □PQR = □PRS

□QPR + 2□TQR = 2□TRS (As QT and RT are angle bisectors)

□QPR = 2(□TRS − □TQR)

□QPR = 2□QTR [By using equation (1)]

□QTR = $\frac{1}{2}$ □QPR

NCERT

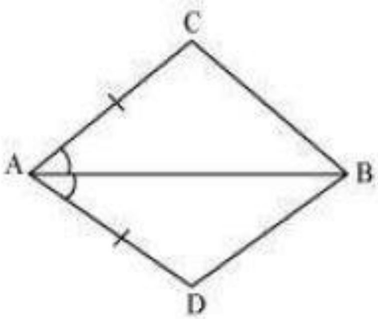
Class 9th Maths

Chapter 7: Triangles

Exercise 7.1

Question 1:

In quadrilateral ACBD, AC = AD and AB bisects ∠A (See the given figure). Show that ΔABC ≅ ΔABD. What can you say about BC and BD?



Answer:

In ΔABC and ΔABD,

AC = AD (Given)

∠CAB = ∠DAB (AB bisects ∠A)

AB = AB (Common)

∴ ΔABC ≅ ΔABD (By SAS congruence rule)

∴ BC = BD (By CPCT)

Therefore, BC and BD are of equal lengths.

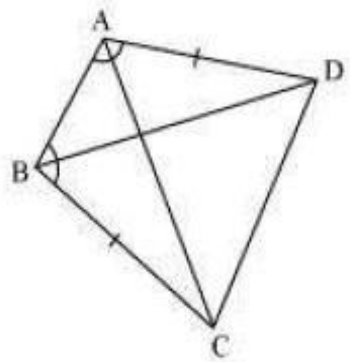
Question 2:

ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (See the given figure). Prove that

(i) ΔABD ≅ ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC.



Answer:

In ΔABD and ΔBAC,

AD = BC (Given)

∠DAB = ∠CBA (Given)

AB = BA (Common)

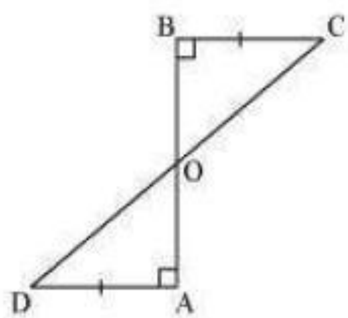
∴ ΔABD ≅ ΔBAC (By SAS congruence rule)

∴ BD = AC (By CPCT)

And, ∠ABD = ∠BAC (By CPCT)

Question 3:

AD and BC are equal perpendiculars to a line segment AB (See the given figure). Show that CD bisects AB.



Answer:

In ΔBOC and ΔAOD,

∠BOC = ∠AOD (Vertically opposite angles)

∠CBO = ∠DAO (Each 90º)

BC = AD (Given)

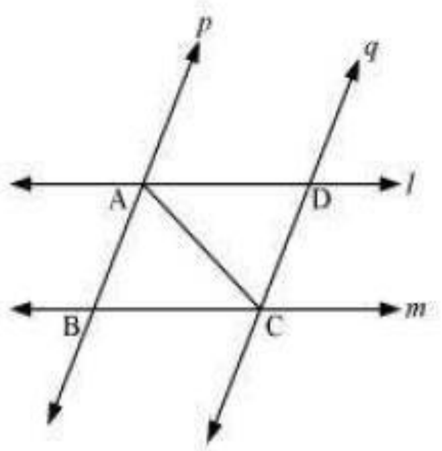
∴ ΔBOC ≅ ΔAOD (AAS congruence rule)

∴ BO = AO (By CPCT)

⇒ CD bisects AB.

Question 4:

l and *m* are two parallel lines intersected by another pair of parallel lines *p* and *q* (see the given figure). Show that ΔABC ≅ ΔCDA.



Answer:

In ΔABC and ΔCDA,

∠BAC = ∠DCA (Alternate interior angles, as *p* || *q*)

AC = CA (Common)

∠BCA = ∠DAC (Alternate interior angles, as *l* || *m*)

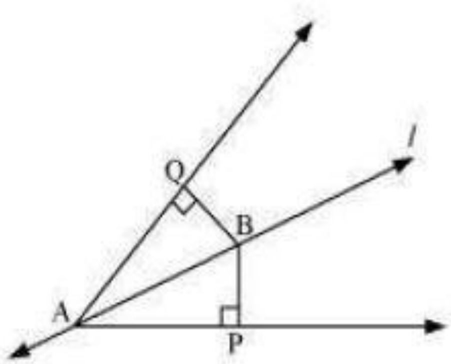
☐ ΔABC ☐ ΔCDA (By ASA congruence rule)

Question 5:

Line *l* is the bisector of an angle ☐A and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of ☐A (see the given figure). Show that:

(i) ΔAPB ☐ ΔAQB

(ii) BP = BQ or B is equidistant from the arms of ☐A.



Answer:

In ΔAPB and ΔAQB,

☐APB = ☐AQB (Each 90º)

☐PAB = ☐QAB (*l* is the angle bisector of ☐A)

AB = AB (Common)

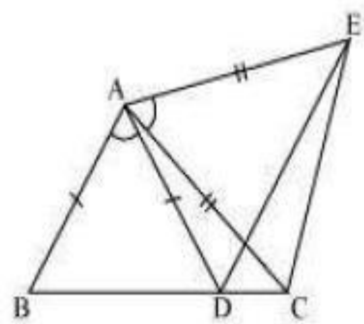
☐ ΔAPB ☐ ΔAQB (By AAS congruence rule)

☐ BP = BQ (By CPCT)

Or, it can be said that B is equidistant from the arms of ☐A.

Question 6:

In the given figure, AC = AE, AB = AD and ☐BAD = ☐EAC. Show that BC = DE.



Answer:

It is given that ☐BAD = ☐EAC

☐BAD + ☐DAC = ☐EAC + ☐DAC

☐BAC = ☐DAE

In ΔBAC and ΔDAE,

AB = AD (Given)

☐BAC = ☐DAE (Proved above)

AC = AE (Given)

☐ ΔBAC ☐ ΔDAE (By SAS congruence rule)

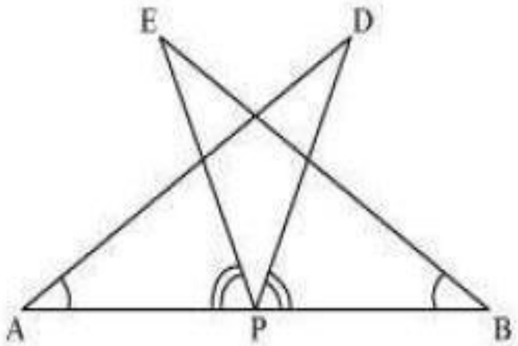
☐ BC = DE (By CPCT)

Question 7:

AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ☐BAD = ☐ABE and ☐EPA = ☐DPB (See the given figure). Show that

(i) ΔDAP ☐ ΔEBP

(ii) AD = BE



Answer:

It is given that □EPA = □DPB

□ □EPA + □DPE = □DPB + □DPE

□ □DPA = □EPB

In Δ DAP and Δ EBP,

□DAP = □EBP (Given)

AP = BP (P is mid-point of AB)

□DPA = □EPB (From above)

□ ΔDAP □ ΔEBP (ASA congruence rule)

□ AD = BE (By CPCT)

Question 8:

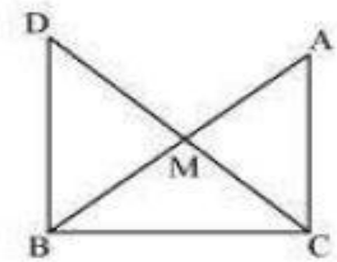
In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see the given figure). Show that:

(i) ΔAMC □ ΔBMD

(ii) □DBC is a right angle.

(iii) ΔDBC □ ΔACB

(iv) CM = $\frac{1}{2}$ AB



Answer:

(i) In ΔAMC and ΔBMD,

AM = BM (M is the mid-point of AB)

□AMC = □BMD (Vertically opposite angles)

CM = DM (Given)

□ ΔAMC □ ΔBMD (By SAS congruence rule)

□ AC = BD (By CPCT)

And, □ACM = □BDM (By CPCT)

(ii) □ACM = □BDM

However, □ACM and □BDM are alternate interior angles.

Since alternate angles are equal,

It can be said that DB || AC

□ □DBC + □ACB = 180º (Co-interior angles)

□ □DBC + 90º = 180º

□ □DBC = 90º

(iii) In ΔDBC and ΔACB,

DB = AC (Already proved)

□DBC = □ACB (Each 90°)

BC = CB (Common)

□ ΔDBC □ ΔACB (SAS congruence rule)

(iv) ΔDBC □ ΔACB

□ AB = DC (By CPCT)

□ AB = 2 CM

□ CM = $\frac{1}{2}$ AB

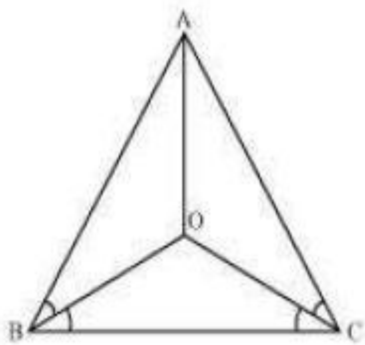
Exercise 7.2

Question 1:

In an isosceles triangle ABC, with AB = AC, the bisectors of □B and □C intersect each other at O. Join A to O. Show that:

(i) OB = OC (ii) AO bisects □A

Answer:



(i) It is given that in triangle ABC, AB = AC

□ □ACB = □ABC (Angles opposite to equal sides of a triangle are equal)

□ $\frac{1}{2}$ □ACB = $\frac{1}{2}$ □ABC

□ □OCB = □OBC

□ OB = OC (Sides opposite to equal angles of a triangle are also equal)

(ii) In ΔOAB and ΔOAC,

AO =AO (Common)

AB = AC (Given)

OB = OC (Proved above)

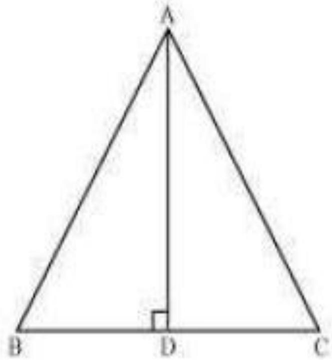
Therefore, ΔOAB □ ΔOAC (By SSS congruence rule)

□ □BAO = □CAO (CPCT)

□ AO bisects □A.

Question 2:

In ΔABC, AD is the perpendicular bisector of BC (see the given figure). Show that ΔABC is an isosceles triangle in which AB = AC.



Answer:

In ΔADC and ΔADB,

AD = AD (Common)

□ADC =□ADB (Each 90º)

CD = BD (AD is the perpendicular bisector of BC)

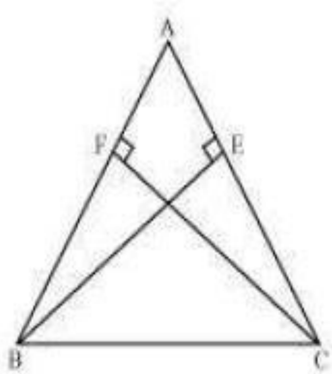
□ ΔADC □ ΔADB (By SAS congruence rule)

□AB = AC (By CPCT)

Therefore, ABC is an isosceles triangle in which AB = AC.

Question 3:

ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see the given figure). Show that these altitudes are equal.



Answer:

In ΔAEB and ΔAFC,

□AEB and □AFC (Each 90º)

□A = □A (Common angle)

AB = AC (Given)

□ ΔAEB □ ΔAFC (By AAS congruence rule)

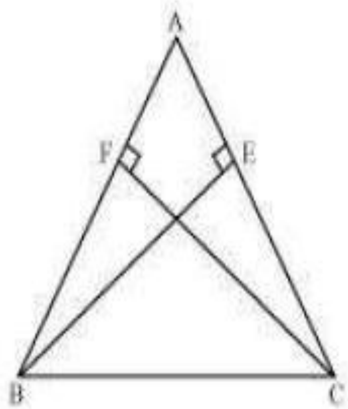
□ BE = CF (By CPCT)

Question 4:

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see the given figure). Show that

(i) ΔABE □ ΔACF

(ii) AB = AC, i.e., ABC is an isosceles triangle.



Answer:

(i) In ΔABE and ΔACF,

□ABE and □ACF (Each 90º)

□A = □A (Common angle)

BE = CF (Given)

□ ΔABE □ ΔACF (By AAS congruence rule)

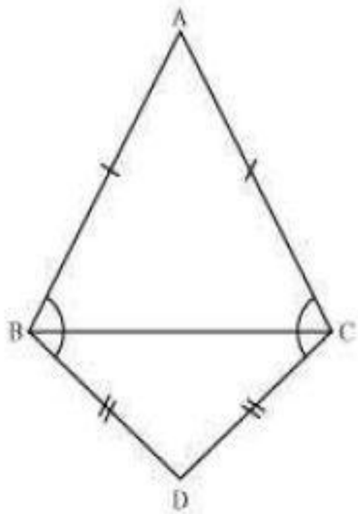
(ii) It has already been proved that

ΔABE □ ΔACF

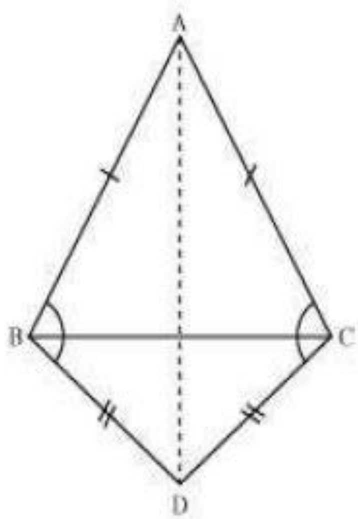
□ AB = AC (By CPCT)

Question 5:

ABC and DBC are two isosceles triangles on the same base BC (see the given figure). Show that □ABD = □ACD.



Answer:



Let us join AD.

In ΔABD and ΔACD,

AB = AC (Given)

BD = CD (Given)

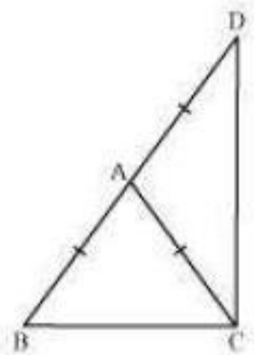
AD = AD (Common side)

☐ ΔABD ≅ ΔACD (By SSS congruence rule)

☐ ☐ABD = ☐ACD (By CPCT)

Question 6:

ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see the given figure). Show that ☐BCD is a right angle.



Answer:

In ΔABC,

AB = AC (Given)

□ □ACB = □ABC (Angles opposite to equal sides of a triangle are also equal)

In ΔACD,

AC = AD

□ □ADC = □ACD (Angles opposite to equal sides of a triangle are also equal)

In ΔBCD,

□ABC + □BCD + □ADC = 180º (Angle sum property of a triangle)

□ □ACB + □ACB +□ACD + □ACD = 180º

□ 2(□ACB + □ACD) = 180º

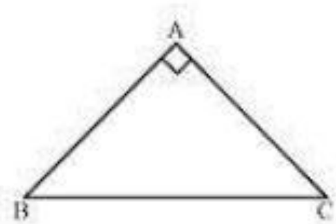
□ 2(□BCD) = 180º

□ □BCD = 90º

Question 7:

ABC is a right angled triangle in which □A = 90º and AB = AC. Find □B and □C.

Answer:



It is given that

AB = AC

□ □C = □B (Angles opposite to equal sides are also equal)

In ΔABC,

□A + □B + □C = 180º (Angle sum property of a triangle)

□ 90º + □B + □C = 180º

□ 90º + □B + □B = 180º

□ 2 □B = 90º

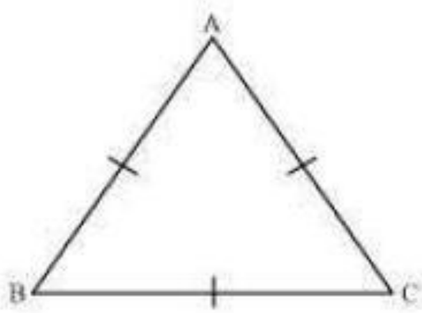
□ □B = 45º

□ □B = □C = 45º

Question 8:

Show that the angles of an equilateral triangle are 60º each.

Answer:



Let us consider that ABC is an equilateral triangle.

Therefore, AB = BC = AC

AB = AC

□ □C = □B (Angles opposite to equal sides of a triangle are equal)

Also,

AC = BC

□ □B = □A (Angles opposite to equal sides of a triangle are equal)

Therefore, we obtain

□A = □B = □C

In ΔABC,

□A + □B + □C = 180°

□ □A + □A + □A = 180°

□ 3□A = 180°

□ □A = 60°

□ □A = □B = □C = 60º

Hence, in an equilateral triangle, all interior angles are of measure 60º.

Exercise 7.3

Question 1:

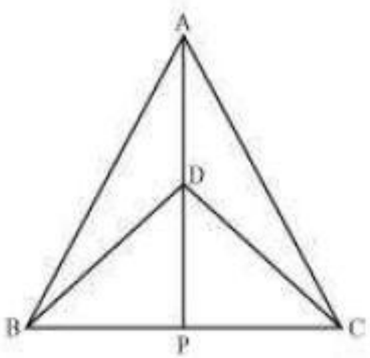
ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see the given figure). If AD is extended to intersect BC at P, show that

(i) ΔABD □ ΔACD

(ii) ΔABP □ ΔACP

(iii) AP bisects □A as well as □D.

(iv) AP is the perpendicular bisector of BC.



Answer:

(i) In ΔABD and ΔACD,

AB = AC (Given)

BD = CD (Given)

AD = AD (Common)

□ ΔABD □ ΔACD (By SSS congruence rule)

□ □BAD = □CAD (By CPCT)

□ □BAP = □CAP .... (1)

(ii) In ΔABP and ΔACP,

AB = AC (Given)

□BAP = □CAP [From equation (1)]

AP = AP (Common)

□ ΔABP □ ΔACP (By SAS congruence rule)

□ BP = CP (By CPCT) ... (2)

(iii) From equation (1),

□BAP = □CAP

Hence, AP bisects □A.

In ΔBDP and ΔCDP,

BD = CD (Given)

DP = DP (Common)

BP = CP [From equation (2)]

□ ΔBDP □ ΔCDP (By S.S.S. Congruence rule)

□ □BDP = □CDP (By CPCT) ... (3)

Hence, AP bisects □D.

(iv) ΔBDP □ ΔCDP

□ □BPD = □CPD (By CPCT) .... (4)

□BPD + □CPD = 180° (Linear pair angles)

□BPD + □BPD = 180°

2□BPD = 180° [From equation (4)]

□BPD = 90° ... (5)

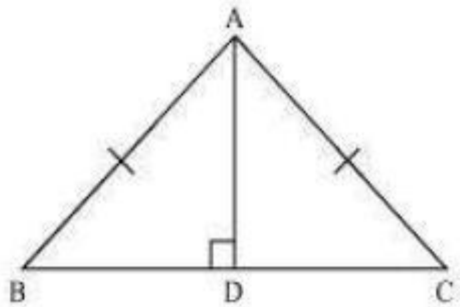
From equations (2) and (5), it can be said that AP is the perpendicular bisector of BC.

Question 2:

AD is an altitude of an isosceles triangles ABC in which AB = AC. Show that

(i) AD bisects BC (ii) AD bisects □A.

Answer:



(i) In ΔBAD and ΔCAD,

□ADB = □ADC (Each 90º as AD is an altitude)

AB = AC (Given)

AD = AD (Common)

□ΔBAD □ ΔCAD (By RHS Congruence rule)

□ BD = CD (By CPCT)

Hence, AD bisects BC.

(ii) Also, by CPCT,

□BAD = □CAD

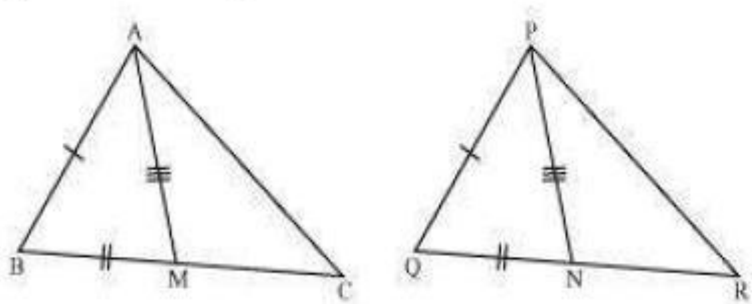
Hence, AD bisects □A.

Question 3:

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see the given figure). Show that:

(i) ΔABM □ ΔPQN

(ii) ΔABC □ ΔPQR



Answer:

(i) In ΔABC, AM is the median to BC.

□ BM = $\frac{1}{2}$ BC

In ΔPQR, PN is the median to QR.

□ QN = $\frac{1}{2}$ QR

However, BC = QR

□ $\frac{1}{2}$ BC = $\frac{1}{2}$ QR

□ BM = QN ... (1)

In ΔABM and ΔPQN,

AB = PQ (Given)

BM = QN [From equation (1)]

AM = PN (Given)

□ ΔABM □ ΔPQN (SSS congruence rule)

□ABM = □PQN (By CPCT)

□ABC = □PQR ... (2)

(ii) In ΔABC and ΔPQR,

AB = PQ (Given)

□ABC = □PQR [From equation (2)]

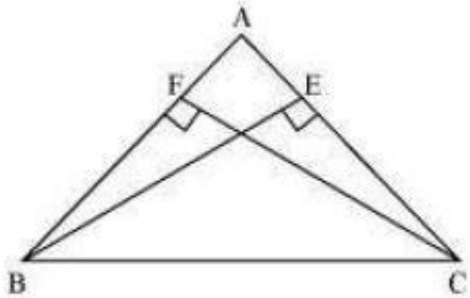
BC = QR (Given)

□ ΔABC □ ΔPQR (By SAS congruence rule)

Question 4:

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Answer:



In ΔBEC and ΔCFB,

☐BEC = ☐CFB (Each 90°)

BC = CB (Common)

BE = CF (Given)

☐ ΔBEC ☐ ΔCFB (By RHS congruency)

☐ ☐BCE = ☐CBF (By CPCT)

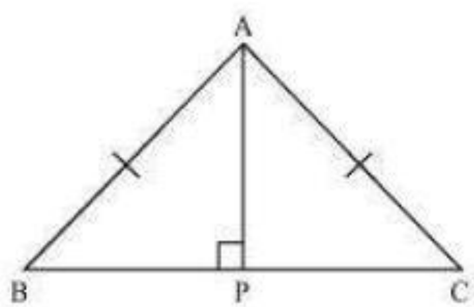
☐ AB = AC (Sides opposite to equal angles of a triangle are equal)

Hence, ΔABC is isosceles.

Question 5:

ABC is an isosceles triangle with AB = AC. Drawn AP ☐ BC to show that ☐B = ☐C.

Answer:



In ΔAPB and ΔAPC,

☐APB = ☐APC (Each 90°)

AB =AC (Given)

AP = AP (Common)

☐ ΔAPB ☐ ΔAPC (Using RHS congruence rule)

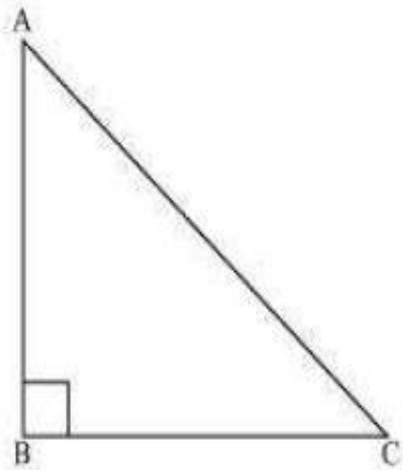
☐ ☐B = ☐C (By using CPCT)

Exercise 7.4

Question 1:

Show that in a right angled triangle, the hypotenuse is the longest side.

Answer:



Let us consider a right-angled triangle ABC, right-angled at B.

In ΔABC,

□A + □B + □C = 180º (Angle sum property of a triangle)

□A + 90º + □C = 180º

□A + □C = 90º

Hence, the other two angles have to be acute (i.e., less than 90º).

□ □B is the largest angle in ΔABC.

□ □B > □A and □B > □C

□ AC > BC and AC > AB

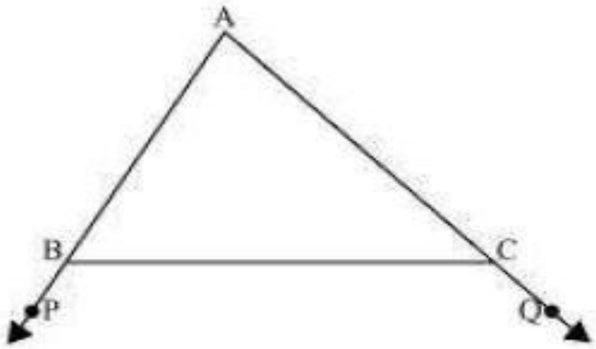
[In any triangle, the side opposite to the larger (greater) angle is longer.]

Therefore, AC is the largest side in ΔABC.

However, AC is the hypotenuse of ΔABC. Therefore, hypotenuse is the longest side in a right-angled triangle.

Question 2:

In the given figure sides AB and AC of ΔABC are extended to points P and Q respectively. Also, □PBC < □QCB. Show that AC > AB.



Answer:

In the given figure,

□ABC + □PBC = 180º (Linear pair)

□ □ABC = 180º – □PBC ... (1)

Also,

□ACB + □QCB = 180º

□ACB = 180º – □QCB … (2)

As □PBC < □QCB,

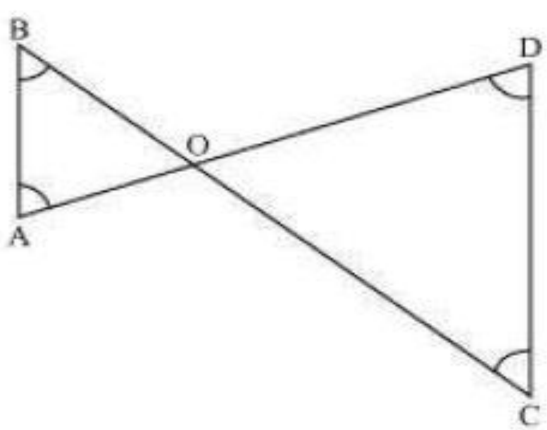
□ 180º – □PBC > 180º – □QCB

□ □ABC > □ACB [From equations (1) and (2)]

□ AC > AB (Side opposite to the larger angle is larger.)

Question 3:

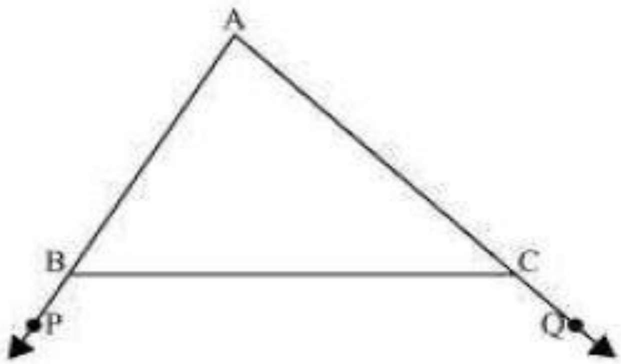
In the given figure, □B < □A and □C < □D. Show that AD < BC.



Answer:

In ΔAOB,

□B < □A



Answer:

In the given figure,

□ABC + □PBC = 180° (Linear pair)

□ □ABC = 180° − □PBC ... (1)

Also,

□ACB + □QCB = 180°

□ACB = 180° − □QCB … (2)

As □PBC < □QCB,

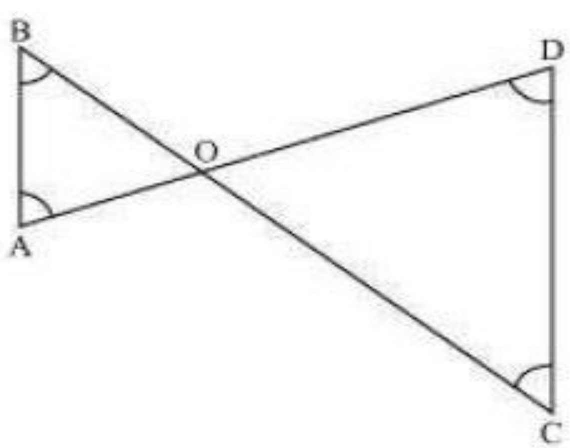
□ 180° − □PBC > 180° − □QCB

□ □ABC > □ACB [From equations (1) and (2)]

□ AC > AB (Side opposite to the larger angle is larger.)

Question 3:

In the given figure, □B < □A and □C < □D. Show that AD < BC.



Answer:

In ΔAOB,

□B < □A

□ AO < BO (Side opposite to smaller angle is smaller) ... (1)

In ΔCOD,

□C < □D

□ OD < OC (Side opposite to smaller angle is smaller) ... (2)

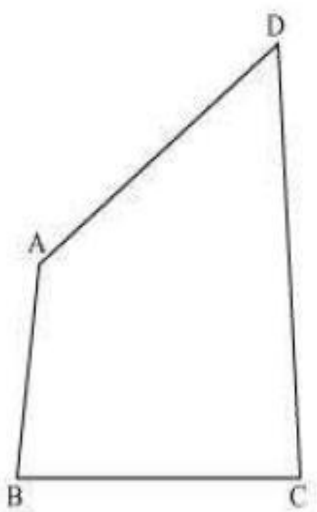
On adding equations (1) and (2), we obtain

AO + OD < BO + OC

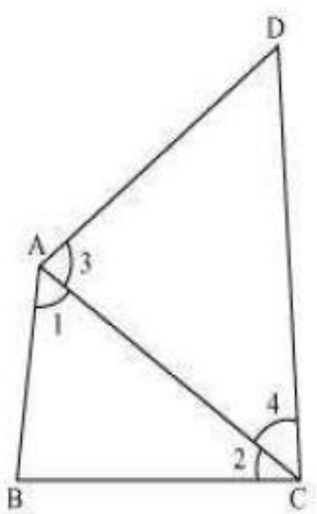
AD < BC

Question 4:

AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see the given figure). Show that □A > □C and □B > □D.



Answer:



Let us join AC.

In ΔABC,

AB < BC (AB is the smallest side of quadrilateral ABCD)

□ □2 < □1 (Angle opposite to the smaller side is smaller) … (1)

In ΔADC,

AD < CD (CD is the largest side of quadrilateral ABCD)

□ □4 < □3 (Angle opposite to the smaller side is smaller) … (2)

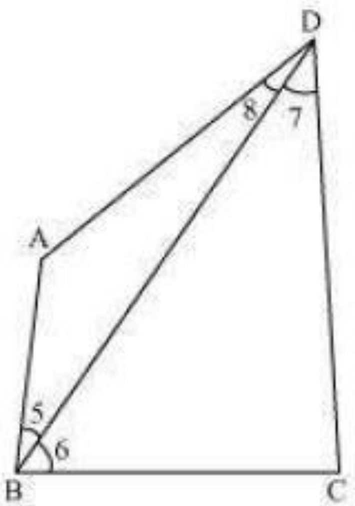
On adding equations (1) and (2), we obtain

□2 + □4 < □1 + □3

□ □C < □A

□ □A > □C

Let us join BD.



In ΔABD,

AB < AD (AB is the smallest side of quadrilateral ABCD)

□ □8 < □5 (Angle opposite to the smaller side is smaller) … (3)

In ΔBDC,

BC < CD (CD is the largest side of quadrilateral ABCD)

□ □7 < □6 (Angle opposite to the smaller side is smaller) … (4)

On adding equations (3) and (4), we obtain

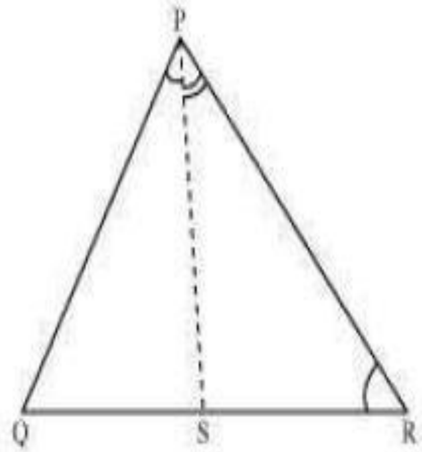
□8 + □7 < □5 + □6

□ □D < □B

□ □B > □D

Question 5:

In the given figure, PR > PQ and PS bisects □QPR. Prove that □PSR >□PSQ.



Answer:

As PR > PQ,

□ □PQR > □PRQ (Angle opposite to larger side is larger) ... (1)

PS is the bisector of □QPR.

□□QPS = □RPS ... (2)

□PSR is the exterior angle of ΔPQS.

□ □PSR = □PQR + □QPS ... (3)

□PSQ is the exterior angle of ΔPRS.

□ □PSQ = □PRQ + □RPS ... (4)

Adding equations (1) and (2), we obtain

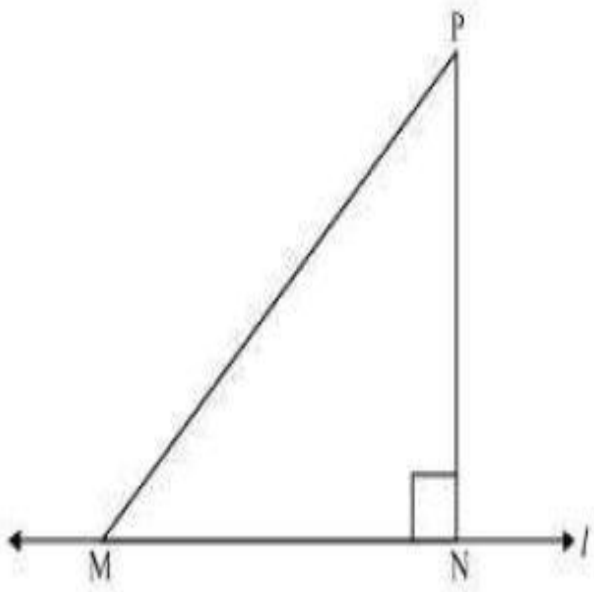
□PQR + □QPS > □PRQ + □RPS

□ □PSR > □PSQ [Using the values of equations (3) and (4)]

Question 6:

Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Answer:



Let us take a line *l* and from point P (i.e., not on line *l*), draw two line segments PN and PM. Let PN be perpendicular to line *l* and PM is drawn at some other angle.

In ΔPNM,

□N = 90°

□P + □N + □M = 180° (Angle sum property of a triangle)

□P + □M = 90°

Clearly, □M is an acute angle.

□ □M < □N

□ PN < PM (Side opposite to the smaller angle is smaller)

Similarly, by drawing different line segments from P to *l*, it can be proved that PN is smaller in comparison to them.

Therefore, it can be observed that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

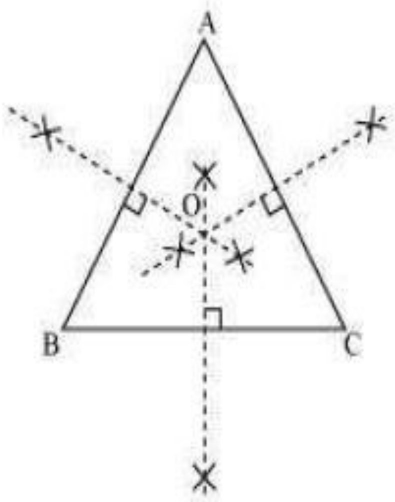
Exercise 7.5

Question 1:

ABC is a triangle. Locate a point in the interior of ΔABC which is equidistant from all the vertices of ΔABC.

Answer:

Circumcentre of a triangle is always equidistant from all the vertices of that triangle. Circumcentre is the point where perpendicular bisectors of all the sides of the triangle meet together.



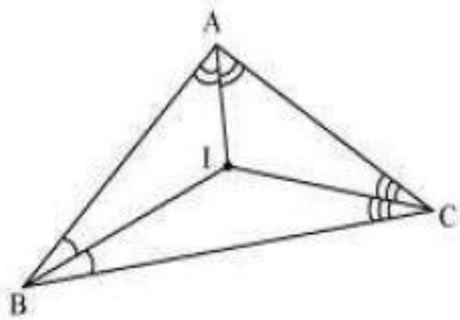
In ΔABC, we can find the circumcentre by drawing the perpendicular bisectors of sides AB, BC, and CA of this triangle. O is the point where these bisectors are meeting together. Therefore, O is the point which is equidistant from all the vertices of ΔABC.

Question 2:

In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Answer:

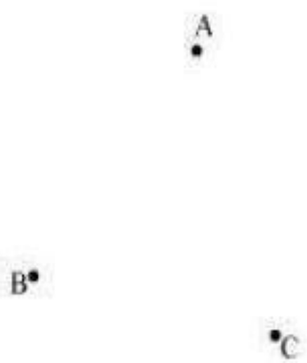
The point which is equidistant from all the sides of a triangle is called the incentre of the triangle. Incentre of a triangle is the intersection point of the angle bisectors of the interior angles of that triangle.



Here, in ΔABC, we can find the incentre of this triangle by drawing the angle bisectors of the interior angles of this triangle. I is the point where these angle bisectors are intersecting each other. Therefore, I is the point equidistant from all the sides of ΔABC.

Question 3:

In a huge park people are concentrated at three points (see the given figure)



A: where there are different slides and swings for children,

B: near which a man-made lake is situated,

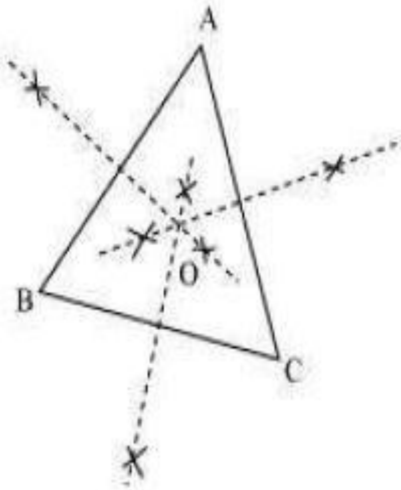
C: which is near to a large parking and exit.

Where should an ice-cream parlour be set up so that maximum number of persons can approach it?

(Hint: The parlor should be equidistant from A, B and C)

Answer:

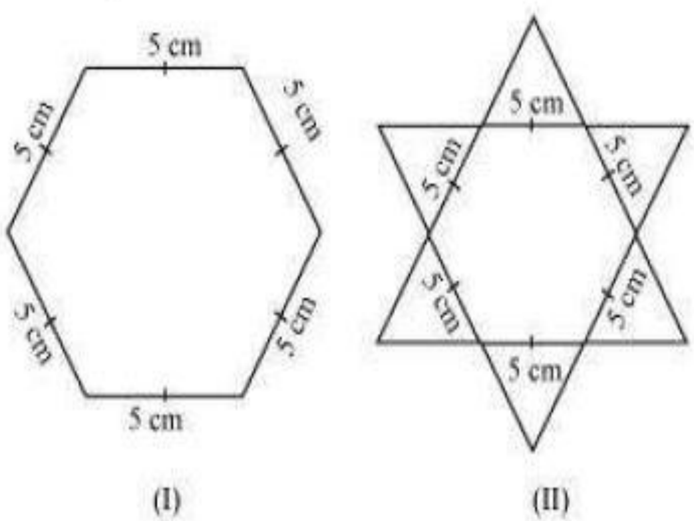
Maximum number of persons can approach the ice-cream parlour if it is equidistant from A, B and C. Now, A, B and C form a triangle. In a triangle, the circumcentre is the only point that is equidistant from its vertices. So, the ice-cream parlour should be set up at the circumcentre O of ΔABC.



In this situation, maximum number of persons can approach it. We can find circumcentre O of this triangle by drawing perpendicular bisectors of the sides of this triangle.

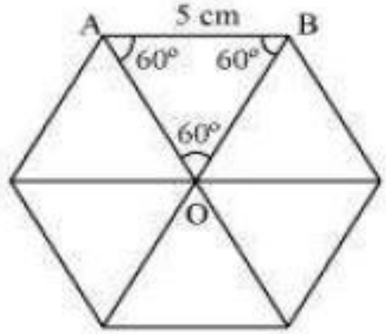
Question 4:

Complete the hexagonal and star shaped *rangolies* (see the given figures) by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?



Answer:

It can be observed that hexagonal-shaped *rangoli* has 6 equilateral triangles in it.



Area of ΔOAB $= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(5)^2$

$= \frac{\sqrt{3}}{4}(25) = \frac{25\sqrt{3}}{4}\text{ cm}^2$

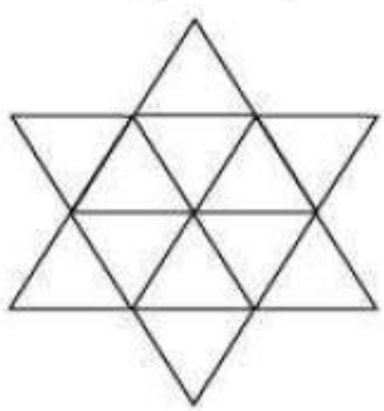
Area of hexagonal-shaped *rangoli* $= 6 \times \frac{25\sqrt{3}}{4} = \frac{75\sqrt{3}}{2}\text{ cm}^2$

Area of equilateral triangle having its side as 1 cm $= \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}\text{ cm}^2$

Number of equilateral triangles of 1 cm side that can be filled

in this hexagonal-shaped *rangoli* $= \frac{\frac{75\sqrt{3}}{2}}{\frac{\sqrt{3}}{4}} = 150$

Star-shaped *rangoli* has 12 equilateral triangles of side 5 cm in it.



Area of star-shaped *rangoli* = $12 \times \frac{\sqrt{3}}{4} \times (5)^2 = 75\sqrt{3}$

Number of equilateral triangles of 1 cm side that can be filled in this star-shaped *rangoli* $= \dfrac{75\sqrt{3}}{\dfrac{\sqrt{3}}{4}} = 300$

Therefore, star-shaped *rangoli* has more equilateral triangles in it.

NCERT
Class 9th Maths
Chapter 8: Quadrilaterals

Exercise 8.1

Question 1:

The angles of quadrilateral are in the ratio 3: 5: 9: 13. Find all the angles of the quadrilateral.

Answer:

Let the common ratio between the angles be x. Therefore, the angles will be $3x$, $5x$, $9x$, and $13x$ respectively.

As the sum of all interior angles of a quadrilateral is 360º,

$\therefore 3x + 5x + 9x + 13x = 360^\circ$

$30x = 360^\circ$

$x = 12^\circ$

Hence, the angles are

$3x = 3 \times 12 = 36^\circ$

$5x = 5 \times 12 = 60^\circ$

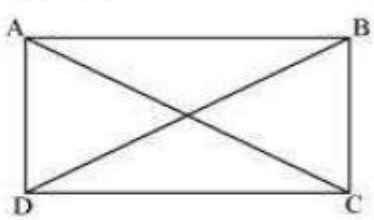
$9x = 9 \times 12 = 108^\circ$

$13x = 13 \times 12 = 156^\circ$

Question 2:

If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Answer:



Let ABCD be a parallelogram. To show that ABCD is a rectangle, we have to prove that one of its interior angles is 90º.

In ΔABC and ΔDCB,

AB = DC (Opposite sides of a parallelogram are equal)

BC = BC (Common)

AC = DB (Given)

∴ $\Delta ABC \cong \Delta DCB$ (By SSS Congruence rule)

⇒ $\angle ABC = \angle DCB$

It is known that the sum of the measures of angles on the same side of transversal is 180º.

$\angle ABC + \angle DCB = 180^\circ$ (AB || CD)

⇒ $\angle ABC + \angle ABC = 180^\circ$

⇒ $2\angle ABC = 180^\circ$

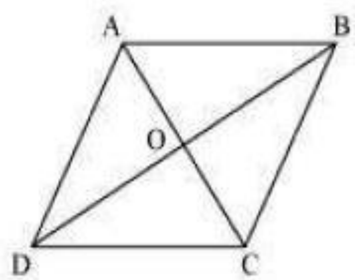
⇒ $\angle ABC = 90^\circ$

Since ABCD is a parallelogram and one of its interior angles is 90º, ABCD is a rectangle.

Question 3:

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Answer:



Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at right angle i.e., OA = OC, OB = OD, and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$. To prove ABCD a rhombus, we have to prove ABCD is a parallelogram and all the sides of ABCD are equal.

In ΔAOD and ΔCOD,

OA = OC (Diagonals bisect each other)

$\angle AOD = \angle COD$ (Given)

OD = OD (Common)

∴ $\Delta AOD \cong \Delta COD$ (By SAS congruence rule)

∴ AD = CD (1)

Similarly, it can be proved that

AD = AB and CD = BC (2)

From equations (1) and (2),

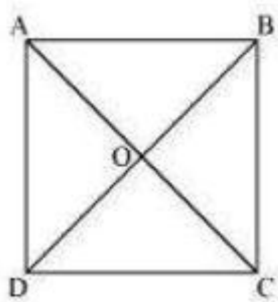
AB = BC = CD = AD

Since opposite sides of quadrilateral ABCD are equal, it can be said that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, it can be said that ABCD is a rhombus.

Question 4:

Show that the diagonals of a square are equal and bisect each other at right angles.

Answer:



Let ABCD be a square. Let the diagonals AC and BD intersect each other at a point O. To prove that the diagonals of a square are equal and bisect each other at right angles, we have to prove AC = BD, OA = OC, OB = OD, and ∠AOB = 90º.

In ΔABC and ΔDCB,

AB = DC (Sides of a square are equal to each other)

∠ABC = ∠DCB (All interior angles are of 90°)

BC = CB (Common side)

∴ ΔABC ≅ ΔDCB (By SAS congruency)

∴ AC = DB (By CPCT)

Hence, the diagonals of a square are equal in length.

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

□ABO = □CDO (Alternate interior angles)

AB = CD (Sides of a square are always equal)

□ ΔAOB □ ΔCOD (By AAS congruence rule)

□ AO = CO and OB = OD (By CPCT)

Hence, the diagonals of a square bisect each other.

In ΔAOB and ΔCOB,

As we had proved that diagonals bisect each other, therefore,

AO = CO

AB = CB (Sides of a square are equal)

BO = BO (Common)

□ ΔAOB □ ΔCOB (By SSS congruency)

□ □AOB = □COB (By CPCT)

However, □AOB + □COB = 180º (Linear pair)

2□AOB = 180º

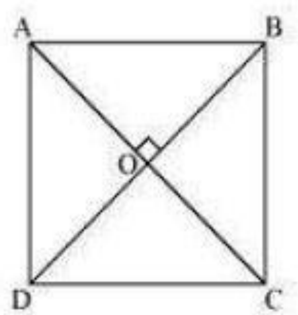
□AOB = 90º

Hence, the diagonals of a square bisect each other at right angles.

Question 5:

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Answer:



Let us consider a quadrilateral ABCD in which the diagonals AC and BD intersect each other at O. It is given that the diagonals of ABCD are equal and bisect each other at right angles. Therefore, AC = BD, OA = OC, OB = OD, and □AOB = □BOC = □COD = □AOD = 90º. To prove ABCD is a square, we have to prove that ABCD is a parallelogram, AB = BC = CD = AD, and one of its interior angles is 90º.

In ΔAOB and ΔCOD,

AO = CO (Diagonals bisect each other)

OB = OD (Diagonals bisect each other)

□AOB = □COD (Vertically opposite angles)

□ ΔAOB □ ΔCOD (SAS congruence rule)

□ AB = CD (By CPCT) ... (1)

And, □OAB = □OCD (By CPCT)

However, these are alternate interior angles for line AB and CD and alternate interior angles are equal to each other only when the two lines are parallel.

□ AB || CD ... (2)

From equations (1) and (2), we obtain

ABCD is a parallelogram.

In ΔAOD and ΔCOD,

AO = CO (Diagonals bisect each other)

□AOD = □COD (Given that each is 90º)

OD = OD (Common)

□ ΔAOD □ ΔCOD (SAS congruence rule)

□ AD = DC ... (3)

However, AD = BC and AB = CD (Opposite sides of parallelogram ABCD)

□ AB = BC = CD = DA

Therefore, all the sides of quadrilateral ABCD are equal to each other.

In ΔADC and ΔBCD,

AD = BC (Already proved)

AC = BD (Given)

DC = CD (Common)

□ ΔADC □ ΔBCD (SSS Congruence rule)

□ □ADC = □BCD (By CPCT)

However, □ADC + □BCD = 180º (Co-interior angles)

□ □ADC + □ADC = 180º

□ 2□ADC = 180º

□ □ADC = 90º

One of the interior angles of quadrilateral ABCD is a right angle.

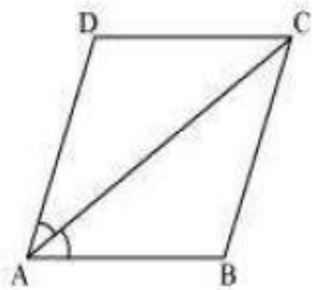
Thus, we have obtained that ABCD is a parallelogram, AB = BC = CD = AD and one of its interior angles is 90º. Therefore, ABCD is a square.

Question 6:

Diagonal AC of a parallelogram ABCD bisects ☐A (see the given figure). Show that

(i) It bisects ☐C also,

(ii) ABCD is a rhombus.



Answer:

(i) ABCD is a parallelogram.

☐ ☐DAC = ☐BCA (Alternate interior angles) ... (1)

And, ☐BAC = ☐DCA (Alternate interior angles) ... (2)

However, it is given that AC bisects ☐A.

☐ ☐DAC = ☐BAC ... (3)

From equations (1), (2), and (3), we obtain

☐DAC = ☐BCA = ☐BAC = ☐DCA ... (4)

☐ ☐DCA = ☐BCA

Hence, AC bisects ☐C.

(ii)From equation (4), we obtain

☐DAC = ☐DCA

☐ DA = DC (Side opposite to equal angles are equal)

However, DA = BC and AB = CD (Opposite sides of a parallelogram)

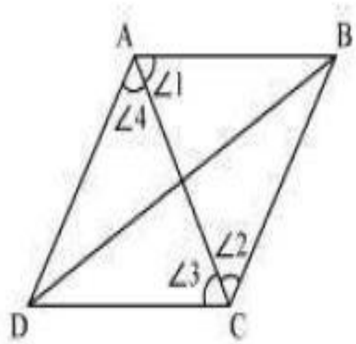
☐ AB = BC = CD = DA

Hence, ABCD is a rhombus.

Question 7:

ABCD is a rhombus. Show that diagonal AC bisects □A as well as □C and diagonal BD bisects □B as well as □D.

Answer:



Let us join AC.

In ΔABC,

BC = AB (Sides of a rhombus are equal to each other)

□ □1 = □2 (Angles opposite to equal sides of a triangle are equal)

However, □1 = □3 (Alternate interior angles for parallel lines AB and CD)

□ □2 = □3

Therefore, AC bisects □C.

Also, □2 = □4 (Alternate interior angles for || lines BC and DA)

□ □1 = □4

Therefore, AC bisects □A.

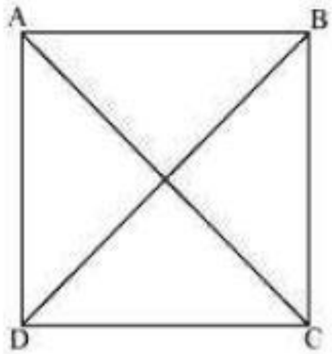
Similarly, it can be proved that BD bisects □B and □D as well.

Question 8:

ABCD is a rectangle in which diagonal AC bisects □A as well as □C. Show that:

(i) ABCD is a square (ii) diagonal BD bisects □B as well as □D.

Answer:



(i) It is given that ABCD is a rectangle.

□□A = □C

$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow \angle DAC = \angle DCA \qquad (AC \text{ bisects } \angle A \text{ and } \angle C)$

CD = DA (Sides opposite to equal angles are also equal)

However, DA = BC and AB = CD (Opposite sides of a rectangle are equal)

□ AB = BC = CD = DA

ABCD is a rectangle and all of its sides are equal.

Hence, ABCD is a square.

(ii) Let us join BD.

In ΔBCD,

BC = CD (Sides of a square are equal to each other)

□CDB = □CBD (Angles opposite to equal sides are equal)

However, □CDB = □ABD (Alternate interior angles for AB || CD)

□ □CBD = □ABD

□ BD bisects ∠B.

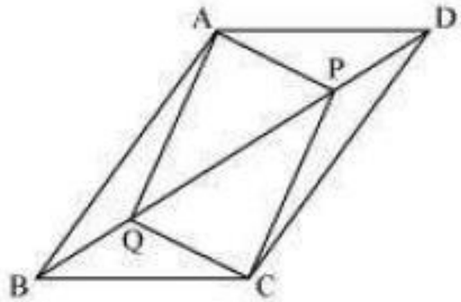
Also, □CBD = □ADB (Alternate interior angles for BC || AD)

□ □CDB = □ABD

□ BD bisects □D.

Question 9:

In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see the given figure). Show that:



(i) ΔAPD □ ΔCQB

(ii) AP = CQ

(iii) ΔAQB □ ΔCPD

(iv) AQ = CP

(v) APCQ is a parallelogram

Answer:

(i) In ΔAPD and ΔCQB,

□ADP = □CBQ (Alternate interior angles for BC || AD)

AD = CB (Opposite sides of parallelogram ABCD)

DP = BQ (Given)

□ ΔAPD □ ΔCQB (Using SAS congruence rule)

(ii) As we had observed that ΔAPD □ ΔCQB,

□ AP = CQ (CPCT)

(iii) In ΔAQB and ΔCPD,

□ABQ = □CDP (Alternate interior angles for AB || CD)

AB = CD (Opposite sides of parallelogram ABCD)

BQ = DP (Given)

□ ΔAQB □ ΔCPD (Using SAS congruence rule)

(iv) As we had observed that ΔAQB □ ΔCPD,

□ AQ = CP (CPCT)

(v) From the result obtained in (ii) and (iv),

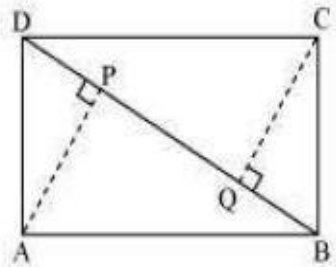
AQ = CP and

AP = CQ

Since opposite sides in quadrilateral APCQ are equal to each other, APCQ is a parallelogram.

Question 10:

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (See the given figure). Show that



(i) ΔAPB □ ΔCQD

(ii) AP = CQ

Answer:

(i) In ΔAPB and ΔCQD,

□APB = □CQD (Each 90°)

AB = CD (Opposite sides of parallelogram ABCD)

□ABP = □CDQ (Alternate interior angles for AB || CD)

□ ΔAPB □ ΔCQD (By AAS congruency)

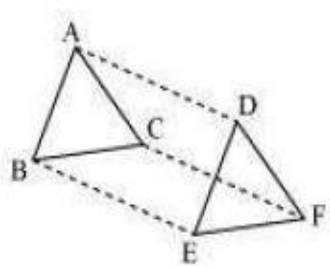
(ii) By using the above result

ΔAPB □ ΔCQD, we obtain

AP = CQ (By CPCT)

Question 11:

In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see the given figure). Show that



(i) Quadrilateral ABED is a parallelogram

(ii) Quadrilateral BEFC is a parallelogram

(iii) AD || CF and AD = CF

(iv) Quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) ΔABC ☐ ΔDEF.

Answer:

(i) It is given that AB = DE and AB || DE.

If two opposite sides of a quadrilateral are equal and parallel to each other, then it will be a parallelogram.

Therefore, quadrilateral ABED is a parallelogram.

(ii) Again, BC = EF and BC || EF

Therefore, quadrilateral BCEF is a parallelogram.

(iii) As we had observed that ABED and BEFC are parallelograms, therefore

AD = BE and AD || BE

(Opposite sides of a parallelogram are equal and parallel)

And, BE = CF and BE || CF

(Opposite sides of a parallelogram are equal and parallel)

☐ AD = CF and AD || CF

(iv) As we had observed that one pair of opposite sides (AD and CF) of quadrilateral ACFD are equal and parallel to each other, therefore, it is a parallelogram.

(v) As ACFD is a parallelogram, therefore, the pair of opposite sides will be equal and parallel to each other.

☐ AC || DF and AC = DF

(vi) ΔABC and ΔDEF,

AB = DE (Given)

BC = EF (Given)

AC = DF (ACFD is a parallelogram)

☐ ΔABC ☐ ΔDEF (By SSS congruence rule)

Question 12:

□ □C = □D

(iii) In ΔABC and ΔBAD,

AB = BA (Common side)

BC = AD (Given)

□B = □A (Proved before)

□ ΔABC □ ΔBAD (SAS congruence rule)

(iv) We had observed that,

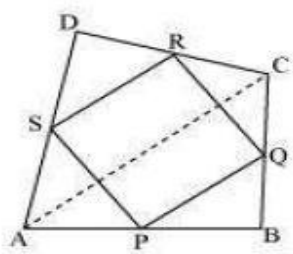
ΔABC □ ΔBAD

□ AC = BD (By CPCT)

Exercise 8.2

Question 1:

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see the given figure). AC is a diagonal. Show that:



(i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Answer:

(i) In ΔADC, S and R are the mid-points of sides AD and CD respectively.

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.

□ SR || AC and SR = $\frac{1}{2}$ AC ... (1)

(ii) In ΔABC, P and Q are mid-points of sides AB and BC respectively. Therefore, by using mid-point theorem,

PQ || AC and PQ = $\frac{1}{2}$ AC ... (2)

Using equations (1) and (2), we obtain

PQ || SR and PQ = SR ... (3)

□ PQ = SR

(iii) From equation (3), we obtained

PQ || SR and PQ = SR

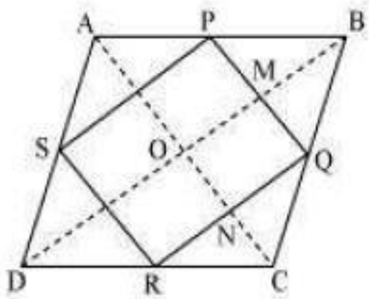
Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal.

Hence, PQRS is a parallelogram.

Question 2:

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Answer:



In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

□ PQ || AC and PQ = $\frac{1}{2}$ AC (Using mid-point theorem) ... (1)

In ΔADC,

R and S are the mid-points of CD and AD respectively.

□ RS || AC and RS = $\frac{1}{2}$ AC (Using mid-point theorem) ... (2)

From equations (1) and (2), we obtain

PQ || RS and PQ = RS

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Let the diagonals of rhombus ABCD intersect each other at point O.

In quadrilateral OMQN,

MQ || ON (∵ PQ || AC)

QN || OM (∵ QR || BD)

Therefore, OMQN is a parallelogram.

□ □MQN = □NOM

□ □PQR = □NOM

However, □NOM = 90° (Diagonals of a rhombus are perpendicular to each other)

☐ ☐PQR = 90º

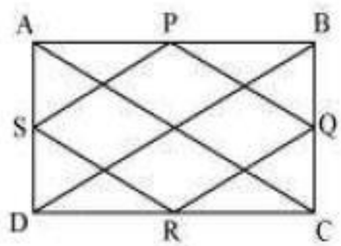
Clearly, PQRS is a parallelogram having one of its interior angles as 90º.

Hence, PQRS is a rectangle.

Question 3:

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Answer:



Let us join AC and BD.

In ΔABC,

P and Q are the mid-points of AB and BC respectively.

☐ PQ || AC and PQ = $\frac{1}{2}$ AC (Mid-point theorem) ... (1)

Similarly in ΔADC,

SR || AC and SR = $\frac{1}{2}$ AC (Mid-point theorem) ... (2)

Clearly, PQ || SR and PQ = SR

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

☐ PS || QR and PS = QR (Opposite sides of parallelogram)... (3)

In ΔBCD, Q and R are the mid-points of side BC and CD respectively.

☐ QR || BD and QR = $\frac{1}{2}$ BD (Mid-point theorem) ... (4)

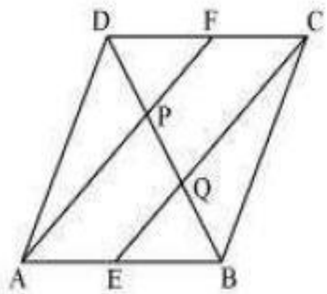
However, the diagonals of a rectangle are equal.

☐ AC = BD ...(5)

By using equation (1), (2), (3), (4), and (5), we obtain

Question 5:

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see the given figure). Show that the line segments AF and EC trisect the diagonal BD.



Answer:

ABCD is a parallelogram.

□AB || CD

And hence, AE || FC

Again, AB = CD (Opposite sides of parallelogram ABCD)

$\frac{1}{2}$ AB = $\frac{1}{2}$ CD

AE = FC (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other. Therefore, AECF is a parallelogram.

□ AF || EC (Opposite sides of a parallelogram)

In ΔDQC, F is the mid-point of side DC and FP || CQ (as AF || EC). Therefore, by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ.

□ DP = PQ ... (1)

Similarly, in ΔAPB, E is the mid-point of side AB and EQ || AP (as AF || EC). Therefore, by using the converse of mid-point theorem, it can be said that

Q is the mid-point of PB.

□ PQ = QB ... (2)

From equations (1) and (2),

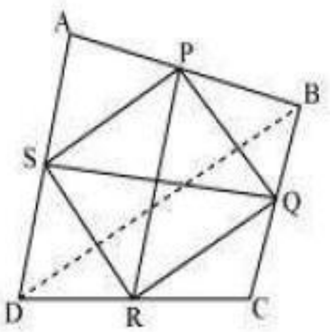
DP = PQ = BQ

Hence, the line segments AF and EC trisect the diagonal BD.

Question 6:

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Answer:



Let ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, and DA respectively. Join PQ, QR, RS, SP, and BD.

In ΔABD, S and P are the mid-points of AD and AB respectively. Therefore, by using mid-point theorem, it can be said that

SP || BD and SP = $\frac{1}{2}$ BD ... (1)

Similarly in ΔBCD,

QR || BD and QR = $\frac{1}{2}$ BD ... (2)

From equations (1) and (2), we obtain

SP || QR and SP = QR

In quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other. Therefore, SPQR is a parallelogram.

We know that diagonals of a parallelogram bisect each other.

Hence, PR and QS bisect each other.

Question 7:

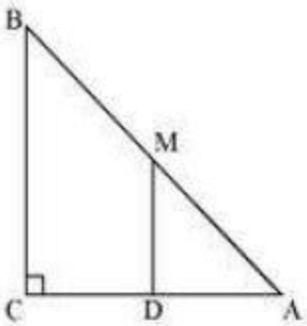
ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) MD □ AC

(iii) $CM = MA = \frac{1}{2} AB$

Answer:



(i) In ΔABC,

It is given that M is the mid-point of AB and MD || BC.

Therefore, D is the mid-point of AC. (Converse of mid-point theorem)

(ii) As DM || CB and AC is a transversal line for them, therefore,

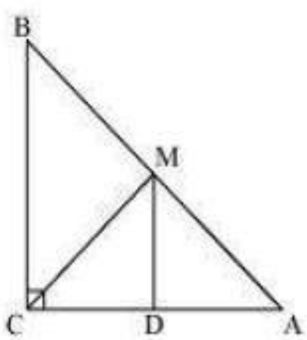
□MDC + □DCB = 180º (Co-interior angles)

□MDC + 90º = 180º

□MDC = 90º

□ MD □ AC

(iii) Join MC.



In ΔAMD and ΔCMD,

AD = CD (D is the mid-point of side AC)

□ADM = □CDM (Each 90º)

DM = DM (Common)

□ΔAMD □ ΔCMD (By SAS congruence rule)

Therefore, AM = CM (By CPCT)

However, AM = $\frac{1}{2}$ AB (M is the mid-point of AB)

Therefore, it can be said that

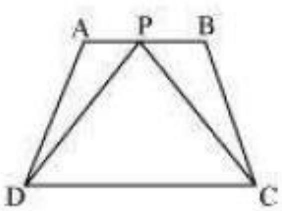
CM = AM = $\frac{1}{2}$ AB

NCERT
Class 9th Maths
Chapter 9: Areas of Parallelograms and Triangles

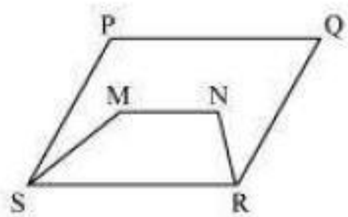
Exercise 9.1

Question 1:

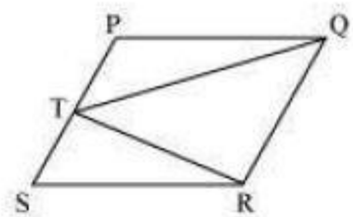
Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



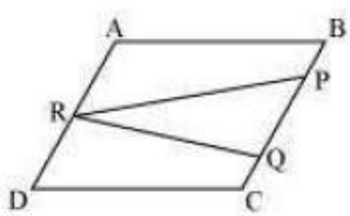
(i)



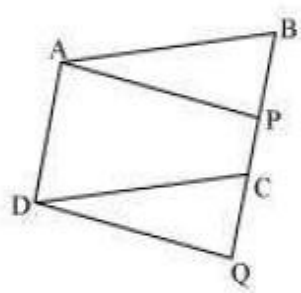
(ii)



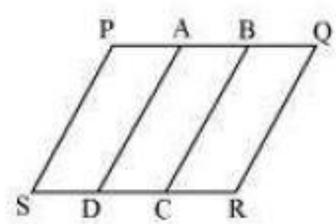
(iii)



(iv)



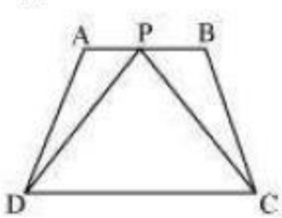
(v)



(vi)

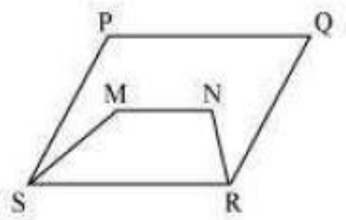
Answer:

(i)



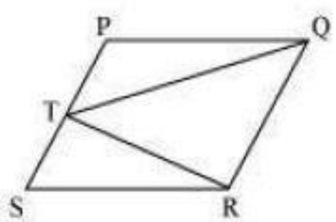
Yes. It can be observed that trapezium ABCD and triangle PCD have a common base CD and these are lying between the same parallel lines AB and CD.

(ii)



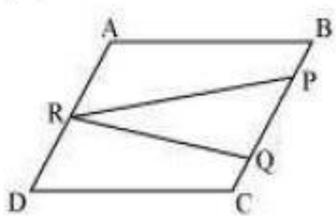
No. It can be observed that parallelogram PQRS and trapezium MNRS have a common base RS. However, their vertices, (i.e., opposite to the common base) P, Q of parallelogram and M, N of trapezium, are not lying on the same line.

(iii)



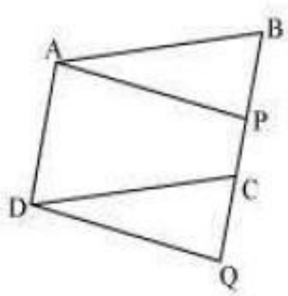
Yes. It can be observed that parallelogram PQRS and triangle TQR have a common base QR and they are lying between the same parallel lines PS and QR.

(iv)



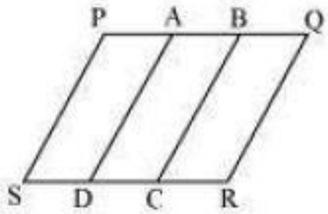
No. It can be observed that parallelogram ABCD and triangle PQR are lying between same parallel lines AD and BC. However, these do not have any common base.

(v)



Yes. It can be observed that parallelogram ABCD and parallelogram APQD have a common base AD and these are lying between the same parallel lines AD and BQ.

(vi)

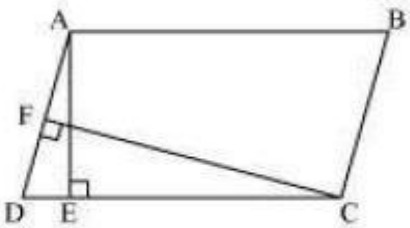


No. It can be observed that parallelogram PBCS and PQRS are lying on the same base PS. However, these do not lie between the same parallel lines.

Exercise 9.2

Question 1:

In the given figure, ABCD is parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Answer:

In parallelogram ABCD, CD = AB = 16 cm

[Opposite sides of a parallelogram are equal]

We know that

Area of a parallelogram = Base × Corresponding altitude

Area of parallelogram ABCD = CD × AE = AD × CF

16 cm × 8 cm = AD × 10 cm

$$AD = \frac{16 \times 8}{10} \text{ cm} = 12.8 \text{ cm}$$

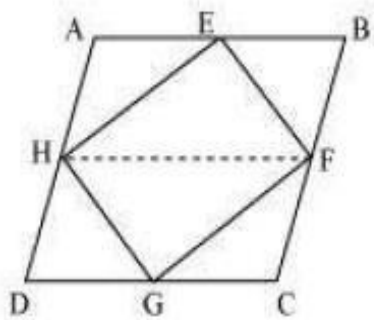
Thus, the length of AD is 12.8 cm.

Question 2:

If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD show that

ar (EFGH) $= \frac{1}{2}$ ar (ABCD)

Answer:



Let us join HF.

In parallelogram ABCD,

AD = BC and AD || BC (Opposite sides of a parallelogram are equal and parallel)

AB = CD (Opposite sides of a parallelogram are equal)

$\Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$ and AH || BF

⇒ AH = BF and AH || BF (∵ H and F are the mid-points of AD and BC)

Therefore, ABFH is a parallelogram.

Since ΔHEF and parallelogram ABFH are on the same base HF and between the same parallel lines AB and HF,

∴ Area (ΔHEF) = $\frac{1}{2}$ Area (ABFH) ... (1)

Similarly, it can be proved that

Area (ΔHGF) = $\frac{1}{2}$ Area (HDCF) ... (2)

On adding equations (1) and (2), we obtain

$$\text{Area}\ (\Delta HEF) + \text{Area}\ (\Delta HGF) = \frac{1}{2}\text{Area}\ (ABFH) + \frac{1}{2}\text{Area}\ (HDCF)$$

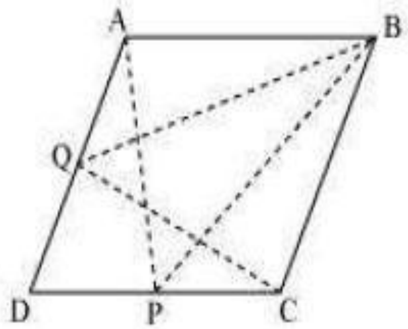
$$= \frac{1}{2}\left[\text{Area}\ (ABFH) + \text{Area}\ (HDCF)\right]$$

$$\Rightarrow \text{Area}\ (EFGH) = \frac{1}{2}\text{Area}\ (ABCD)$$

Question 3:

P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

Answer:



It can be observed that ΔBQC and parallelogram ABCD lie on the same base BC and these are between the same parallel lines AD and BC.

$\therefore$ Area (ΔBQC) = $\frac{1}{2}$ Area (ABCD) ... (1)

Similarly, ΔAPB and parallelogram ABCD lie on the same base AB and between the same parallel lines AB and DC.

$\therefore$ Area (ΔAPB) = $\frac{1}{2}$ Area (ABCD) ... (2)

From equation (1) and (2), we obtain

Area (ΔBQC) = Area (ΔAPB)

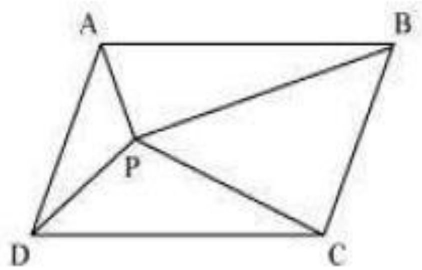
Question 4:

In the given figure, P is a point in the interior of a parallelogram ABCD. Show that

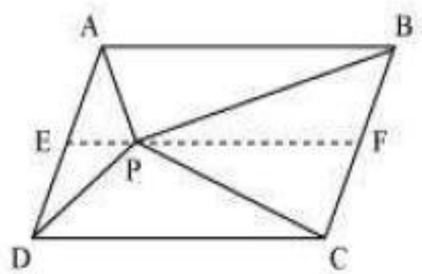
(i) ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

[Hint: Through. P, draw a line parallel to AB]



Answer:



(i) Let us draw a line segment EF, passing through point P and parallel to line segment AB.

In parallelogram ABCD,

AB || EF (By construction) ... (1)

ABCD is a parallelogram.

∴ AD || BC (Opposite sides of a parallelogram)

⇒ AE || BF ... (2)

From equations (1) and (2), we obtain

AB || EF and AE || BF

Therefore, quadrilateral ABFE is a parallelogram.

It can be observed that ΔAPB and parallelogram ABFE are lying on the same base AB and between the same parallel lines AB and EF.

∴ Area (ΔAPB) = $\frac{1}{2}$ Area (ABFE) ... (3)

Similarly, for ΔPCD and parallelogram EFCD,

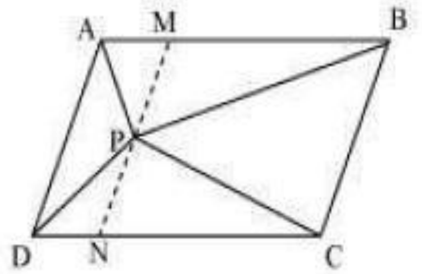
Area (ΔPCD) = $\frac{1}{2}$ Area (EFCD) ... (4)

Adding equations (3) and (4), we obtain

$$\text{Area}(\Delta APB)+\text{Area}(\Delta PCD)=\frac{1}{2}\left[\text{Area}(ABFE)+\text{Area}(EFCD)\right]$$

$$\text{Area}(\Delta APB)+\text{Area}(\Delta PCD)=\frac{1}{2}\text{Area}(ABCD) \quad \ldots (5)$$

(ii)



Let us draw a line segment MN, passing through point P and parallel to line segment AD.

In parallelogram ABCD,

MN || AD (By construction) ... (6)

ABCD is a parallelogram.

∴ AB || DC (Opposite sides of a parallelogram)

⇒ AM || DN ... (7)

From equations (6) and (7), we obtain

MN || AD and AM || DN

Therefore, quadrilateral AMND is a parallelogram.

It can be observed that ΔAPD and parallelogram AMND are lying on the same base AD and between the same parallel lines AD and MN.

∴ Area (ΔAPD) = $\frac{1}{2}$ Area (AMND) ... (8)

Similarly, for ΔPCB and parallelogram MNCB,

Area (ΔPCB) = $\frac{1}{2}$ Area (MNCB) ... (9)

Adding equations (8) and (9), we obtain

$$\text{Area}(\Delta APD) + \text{Area}(\Delta PCB) = \frac{1}{2}\left[\text{Area}(AMND) + \text{Area}(MNCB)\right]$$

$$\text{Area}(\Delta APD) + \text{Area}(\Delta PCB) = \frac{1}{2}\text{Area}(ABCD) \qquad ...(10)$$

On comparing equations (5) and (10), we obtain

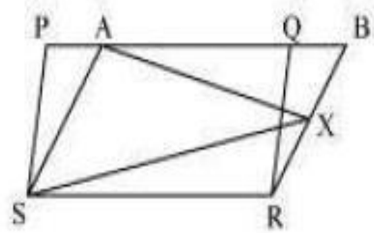
Area (ΔAPD) + Area (ΔPBC) = Area (ΔAPB) + Area (ΔPCD)

Question 5:

In the given figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

(i) ar (PQRS) = ar (ABRS)

(ii) ar (ΔPXS) = $\frac{1}{2}$ ar (PQRS)



Answer:

(i) It can be observed that parallelogram PQRS and ABRS lie on the same base SR and also, these lie in between the same parallel lines SR and PB.

∴ Area (PQRS) = Area (ABRS) ... (1)

(ii) Consider ΔAXS and parallelogram ABRS.

As these lie on the same base and are between the same parallel lines AS and BR,

∴ Area (ΔAXS) = $\frac{1}{2}$ Area (ABRS) ... (2)

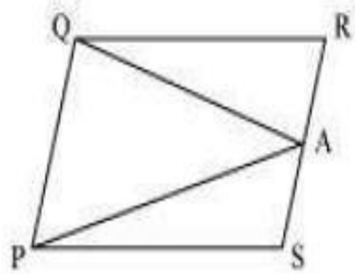
From equations (1) and (2), we obtain

Area (ΔAXS) = $\frac{1}{2}$ Area (PQRS)

Question 6:

A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Answer:



From the figure, it can be observed that point A divides the field into three parts. These parts are triangular in shape – ΔPSA, ΔPAQ, and ΔQRA

Area of ΔPSA + Area of ΔPAQ + Area of ΔQRA = Area of $\| gm$ PQRS ... (1)

We know that if a parallelogram and a triangle are on the same base and between the same parallels, then the area of the triangle is half the area of the parallelogram.

∴ Area (ΔPAQ) = $\frac{1}{2}$ Area (PQRS) ... (2)

From equations (1) and (2), we obtain

Area (ΔPSA) + Area (ΔQRA) = $\frac{1}{2}$ Area (PQRS) ... (3)

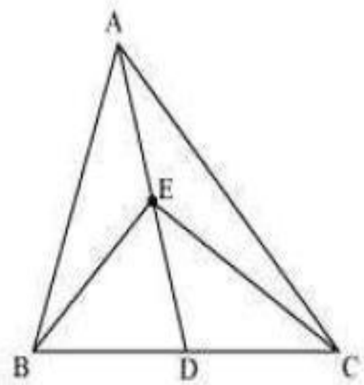
Clearly, it can be observed that the farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and pulses in triangular parts PAQ.

Exercise 9.3

Question 1:

In the given figure, E is any point on median AD of a ΔABC. Show that

ar (ABE) = ar (ACE)



Answer:

AD is the median of ΔABC. Therefore, it will divide ΔABC into two triangles of equal areas.

∴ Area (ΔABD) = Area (ΔACD) ... (1)

ED is the median of ΔEBC.

∴ Area (ΔEBD) = Area (ΔECD) ... (2)

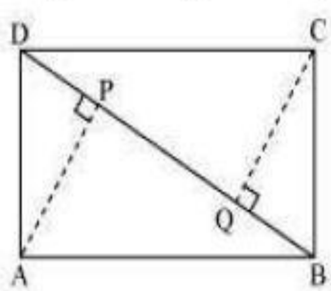
On subtracting equation (2) from equation (1), we obtain

Area (ΔABD) − Area (EBD) = Area (ΔACD) − Area (ΔECD)

Area (ΔABE) = Area (ΔACE)

Question 10:

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (See the given figure). Show that



(i) ΔAPB ≅ ΔCQD

(ii) AP = CQ

Answer:

(i) In ΔAPB and ΔCQD,

∠APB = ∠CQD (Each 90°)

AB = CD (Opposite sides of parallelogram ABCD)

∠ABP = ∠CDQ (Alternate interior angles for AB || CD)

∴ ΔAPB ≅ ΔCQD (By AAS congruency)

(ii) By using the above result

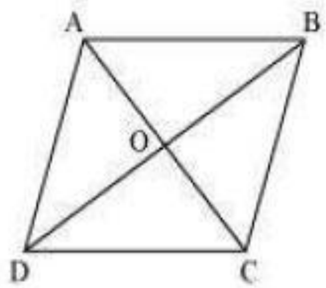
ΔAPB ≅ ΔCQD, we obtain

AP = CQ (By CPCT)

Question 3:

Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Answer:



We know that diagonals of parallelogram bisect each other.

Therefore, O is the mid-point of AC and BD.

BO is the median in ΔABC. Therefore, it will divide it into two triangles of equal areas.

∴ Area (ΔAOB) = Area (ΔBOC) ... (1)

In ΔBCD, CO is the median.

∴ Area (ΔBOC) = Area (ΔCOD) ... (2)

Similarly, Area (ΔCOD) = Area (ΔAOD) ... (3)

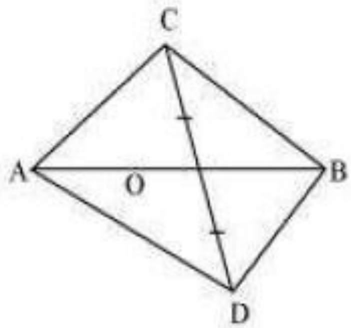
From equations (1), (2), and (3), we obtain

Area (ΔAOB) = Area (ΔBOC) = Area (ΔCOD) = Area (ΔAOD)

Therefore, it is evident that the diagonals of a parallelogram divide it into four triangles of equal area.

Question 4:

In the given figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).



Answer:

Consider ΔACD.

Line-segment CD is bisected by AB at O. Therefore, AO is the median of ΔACD.

∴ Area (ΔACO) = Area (ΔADO) ... (1)

Considering ΔBCD, BO is the median.

∴ Area (ΔBCO) = Area (ΔBDO) ... (2)

Adding equations (1) and (2), we obtain

Area (ΔACO) + Area (ΔBCO) = Area (ΔADO) + Area (ΔBDO)

⇒ Area (ΔABC) = Area (ΔABD)

Question 6:

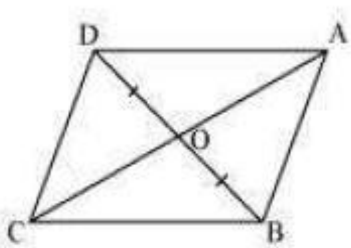
In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

(i) ar (DOC) = ar (AOB)

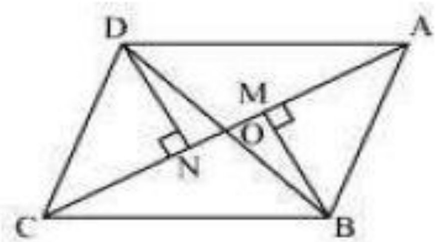
(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.

[Hint: From D and B, draw perpendiculars to AC.]



Answer:



Let us draw DN □ AC and BM □ AC.

(i) In ΔDON and ΔBOM,

□DNO = □BMO (By construction)

□DON = □BOM (Vertically opposite angles)

OD = OB (Given)

By AAS congruence rule,

ΔDON □ ΔBOM

□ DN = BM ... (1)

We know that congruent triangles have equal areas.

□ Area (ΔDON) = Area (ΔBOM) ... (2)

In ΔDNC and ΔBMA,

□DNC = □BMA (By construction)

CD = AB (Given)

DN = BM [Using equation (1)]

□ ΔDNC □ ΔBMA (RHS congruence rule)

□ Area (ΔDNC) = Area (ΔBMA) ... (3)

On adding equations (2) and (3), we obtain

Area (ΔDON) + Area (ΔDNC) = Area (ΔBOM) + Area (ΔBMA)

Therefore, Area (ΔDOC) = Area (ΔAOB)

(ii) We obtained,

Area (ΔDOC) = Area (ΔAOB)

□ Area (ΔDOC) + Area (ΔOCB) = Area (ΔAOB) + Area (ΔOCB)

(Adding Area (ΔOCB) to both sides)

□ Area (ΔDCB) = Area (ΔACB)

(iii) We obtained,

Area (ΔDCB) = Area (ΔACB)

If two triangles have the same base and equal areas, then these will lie between the same parallels.

☐ DA || CB ... (4)

In quadrilateral ABCD, one pair of opposite sides is equal (AB = CD) and the other pair of opposite sides is parallel (DA || CB).

Therefore, ABCD is a parallelogram.

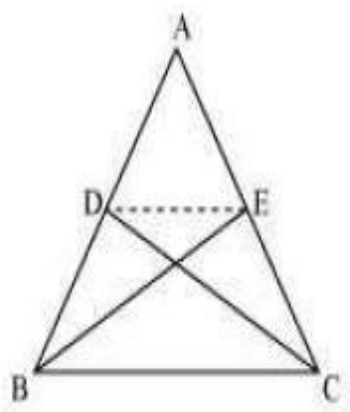
Question 7:

D and E are points on sides AB and AC respectively of ΔABC such that

ar (DBC) = ar (EBC). Prove that DE || BC.

Answer:

Answer:



Since ΔBCE and ΔBCD are lying on a common base BC and also have equal areas, ΔBCE and ΔBCD will lie between the same parallel lines.

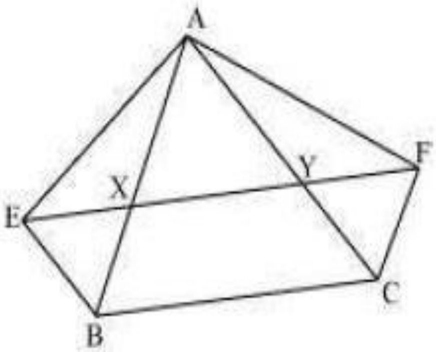
☐ DE || BC

Question 8:

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and E respectively, show that

ar (ABE) = ar (ACF)

Answer:



It is given that

XY || BC ☐ EY || BC

BE || AC ☐ BE || CY

Therefore, EBCY is a parallelogram.

It is given that

XY || BC ☐ XF || BC

FC || AB ☐ FC || XB

Therefore, BCFX is a parallelogram.

Parallelograms EBCY and BCFX are on the same base BC and between the same parallels BC and EF.

☐ Area (EBCY) = Area (BCFX) ... (1)

Consider parallelogram EBCY and ΔAEB

These lie on the same base BE and are between the same parallels BE and AC.

☐ Area (ΔABE) = $\frac{1}{2}$ Area (EBCY) ... (2)

Also, parallelogram BCFX and ΔACF are on the same base CF and between the same parallels CF and AB.

☐ Area (ΔACF) = $\frac{1}{2}$ Area (BCFX) ... (3)

From equations (1), (2), and (3), we obtain

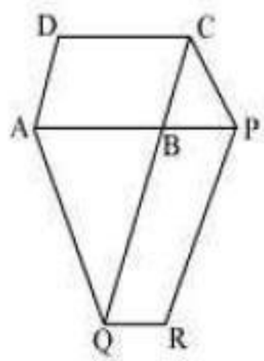
Area (ΔABE) = Area (ΔACF)

Question 9:

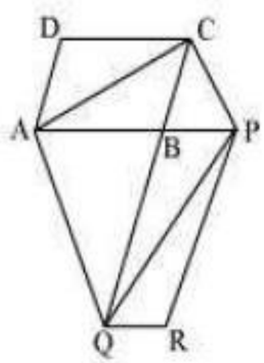
The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that

ar (ABCD) = ar (PBQR).

[Hint: Join AC and PQ. Now compare area (ACQ) and area (APQ)]



Answer:



Let us join AC and PQ.

ΔACQ and ΔAQP are on the same base AQ and between the same parallels AQ and CP.

☐ Area (ΔACQ) = Area (ΔAPQ)

☐ Area (ΔACQ) − Area (ΔABQ) = Area (ΔAPQ) − Area (ΔABQ)

☐ Area (ΔABC) = Area (ΔQBP) … (1)

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively,

☐ Area (ΔABC) = $\frac{1}{2}$ Area (ABCD) … (2)

Area (ΔQBP) = $\frac{1}{2}$ Area (PBQR) ... (3)

From equations (1), (2), and (3), we obtain

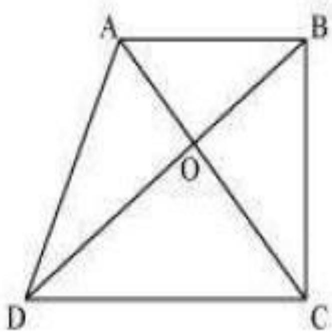
$\frac{1}{2}$ Area (ABCD) = $\frac{1}{2}$ Area (PBQR)

Area (ABCD) = Area (PBQR)

Question 10:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).

Answer:



It can be observed that ΔDAC and ΔDBC lie on the same base DC and between the same parallels AB and CD.

☐ Area (ΔDAC) = Area (ΔDBC)

☐ Area (ΔDAC) − Area (ΔDOC) = Area (ΔDBC) − Area (ΔDOC)

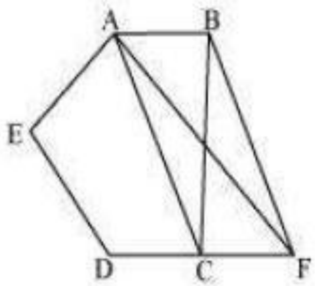
☐ Area (ΔAOD) = Area (ΔBOC)

Question 11:

In the given figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)



Answer:

(i) ΔACB and ΔACF lie on the same base AC and are between

The same parallels AC and BF.

☐ Area (ΔACB) = Area (ΔACF)

(ii) It can be observed that

Area (ΔACB) = Area (ΔACF)

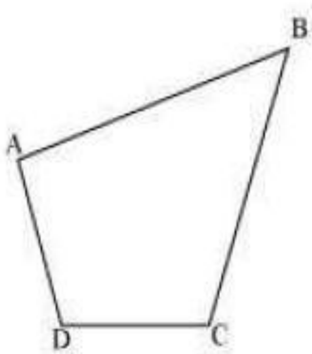
☐ Area (ΔACB) + Area (ACDE) = Area (ACF) + Area (ACDE)

☐ Area (ABCDE) = Area (AEDF)

Question 12:

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Answer:

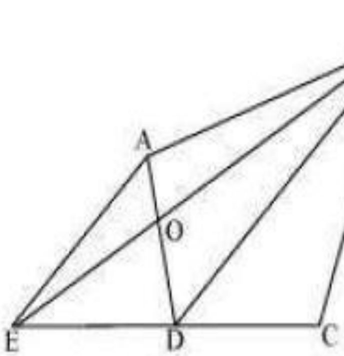


Let quadrilateral ABCD be the original shape of the field.

The proposal may be implemented as follows.

Join diagonal BD and draw a line parallel to BD through point A. Let it meet the extended side CD of ABCD at point E. Join BE and AD. Let them intersect each other at O. Then, portion ΔAOB can be cut from the original field so that the new shape of the field will be ΔBCE. (See figure)

We have to prove that the area of ΔAOB (portion that was cut so as to construct Health Centre) is equal to the area of ΔDEO (portion added to the field so as to make the area of the new field so formed equal to the area of the original field)



It can be observed that ΔDEB and ΔDAB lie on the same base BD and are between the same parallels BD and AE.

☐ Area (ΔDEB) = Area (ΔDAB)

☐ Area (ΔDEB) − Area (ΔDOB) = Area (ΔDAB) − Area (ΔDOB)

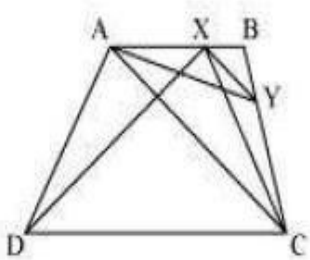
☐ Area (ΔDEO) = Area (ΔAOB)

Question 13:

ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).

[Hint: Join CX.]

Answer:



It can be observed that ΔADX and ΔACX lie on the same base AX and are between the same parallels AB and DC.

☐ Area (ΔADX) = Area (ΔACX) ... (1)

ΔACY and ΔACX lie on the same base AC and are between the same parallels AC and XY.

☐ Area (ΔACY) = Area (ACX) ... (2)

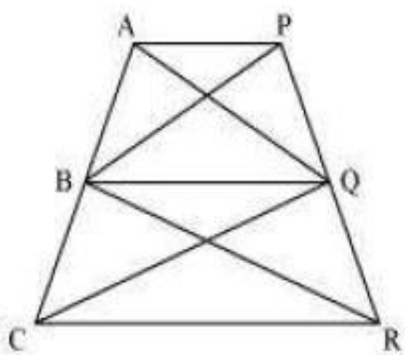
From equations (1) and (2), we obtain

Area (ΔADX) = Area (ΔACY)

Question 14:

In the given figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).

Answer:



Since ΔABQ and ΔPBQ lie on the same base BQ and are between the same parallels AP and BQ,

☐ Area (ΔABQ) = Area (ΔPBQ) ... (1)

Again, ΔBCQ and ΔBRQ lie on the same base BQ and are between the same parallels BQ and CR.

☐ Area (ΔBCQ) = Area (ΔBRQ) ... (2)

On adding equations (1) and (2), we obtain

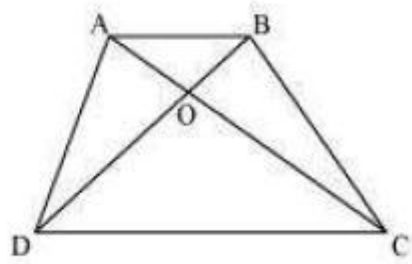
Area (ΔABQ) + Area (ΔBCQ) = Area (ΔPBQ) + Area (ΔBRQ)

☐ Area (ΔAQC) = Area (ΔPBR)

Question 15:

Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

Answer:



It is given that

Area (ΔAOD) = Area (ΔBOC)

Area (ΔAOD) + Area (ΔAOB) = Area (ΔBOC) + Area (ΔAOB)

Area (ΔADB) = Area (ΔACB)

We know that triangles on the same base having areas equal to each other lie between the same parallels.

Therefore, these triangles, ΔADB and ΔACB, are lying between the same parallels.

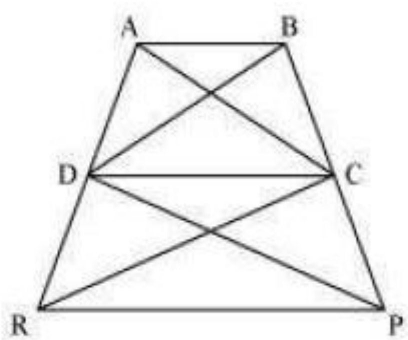
i.e., AB || CD

Therefore, ABCD is a trapezium.

Question 16:

In the given figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.

Answer:



It is given that

Area (ΔDRC) = Area (ΔDPC)

As ΔDRC and ΔDPC lie on the same base DC and have equal areas, therefore, they must lie between the same parallel lines.

☐ DC || RP

Therefore, DCPR is a trapezium.

It is also given that

Area (ΔBDP) = Area (ΔARC)

☐ Area (BDP) − Area (ΔDPC) = Area (ΔARC) − Area (ΔDRC)

☐ Area (ΔBDC) = Area (ΔADC)

Since ΔBDC and ΔADC are on the same base CD and have equal areas, they must lie between the same parallel lines.

☐ AB || CD

Therefore, ABCD is a trapezium.

Exercise 9.4

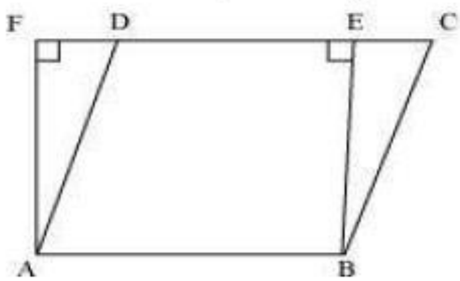
Question 1:

Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Answer:

As the parallelogram and the rectangle have the same base and equal area, therefore, these will also lie between the same parallels.

Consider the parallelogram ABCD and rectangle ABEF as follows.



Here, it can be observed that parallelogram ABCD and rectangle ABEF are between the same parallels AB and CF.

We know that opposite sides of a parallelogram or a rectangle are of equal lengths. Therefore,

AB = EF (For rectangle)

AB = CD (For parallelogram)

☐ CD = EF

☐ AB + CD = AB + EF ... (1)

Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.

☐ AF < AD

And similarly, BE < BC

☐ AF + BE < AD + BC ... (2)

From equations (1) and (2), we obtain

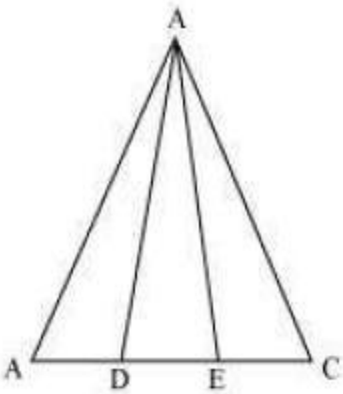
AB + EF + AF + BE < AD + BC + AB + CD

Perimeter of rectangle ABEF < Perimeter of parallelogram ABCD

Question 2:

In the following figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).

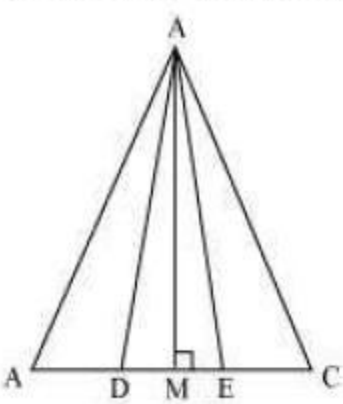
Can you answer the question that you have left in the 'Introduction' of this chapter, whether the field of *Budhia* has been actually divided into three parts of equal area?



[**Remark:** Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into *n* equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into *n* triangles of equal areas.]

Answer:

Let us draw a line segment AM □ BC.



We know that,

Area of a triangle $= \frac{1}{2} \times$ Base × Altitude

$$\text{Area}\left(\Delta ADE\right)=\frac{1}{2}\times DE\times AM$$

$$\text{Area}\left(\Delta ABD\right)=\frac{1}{2}\times BD\times AM$$

$$\text{Area}\left(\Delta AEC\right)=\frac{1}{2}\times EC\times AM$$

It is given that DE = BD = EC

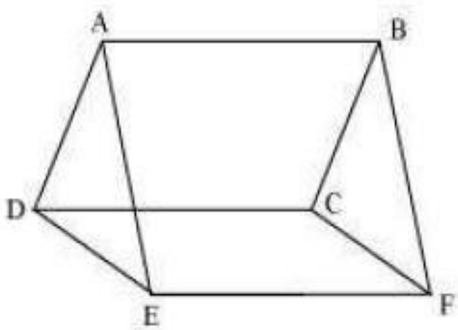
□ $\frac{1}{2}\times DE\times AM=\frac{1}{2}\times BD\times AM=\frac{1}{2}\times EC\times AM$

□ Area (ΔADE) = Area (ΔABD) = Area (ΔAEC)

It can be observed that *Budhia* has divided her field into 3 equal parts.

Question 3:

In the following figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Answer:

It is given that ABCD is a parallelogram. We know that opposite sides of a parallelogram are equal.

□ AD = BC ... (1)

Similarly, for parallelograms DCEF and ABFE, it can be proved that

DE = CF ... (2)

And, EA = FB ... (3)

In ΔADE and ΔBCF,

AD = BC [Using equation (1)]

DE = CF [Using equation (2)]

EA = FB [Using equation (3)]

□ ΔADE □ BCF (SSS congruence rule)

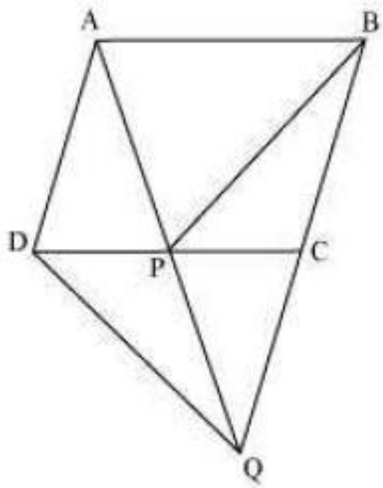
□ Area (ΔADE) = Area (ΔBCF)

Question 4:

In the following figure, ABCD is parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that

ar (BPC) = ar (DPQ).

[Hint: Join AC.]

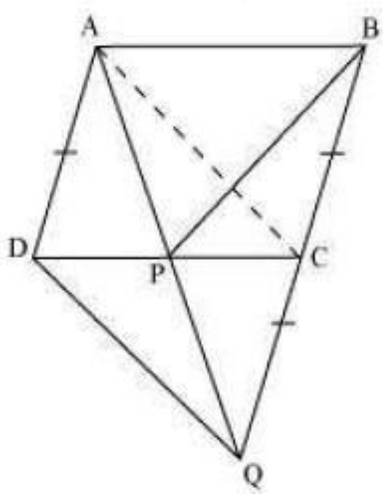


Answer:

It is given that ABCD is a parallelogram.

AD || BC and AB || DC(Opposite sides of a parallelogram are parallel to each other)

Join point A to point C.



Consider ΔAPC and ΔBPC

ΔAPC and ΔBPC are lying on the same base PC and between the same parallels PC and AB. Therefore,

Area (ΔAPC) = Area (ΔBPC) ... (1)

In quadrilateral ACDQ, it is given that

AD = CQ

Since ABCD is a parallelogram,

AD || BC (Opposite sides of a parallelogram are parallel)

CQ is a line segment which is obtained when line segment BC is produced.

□ AD || CQ

We have,

AC = DQ and AC || DQ

Hence, ACQD is a parallelogram.

Consider ΔDCQ and ΔACQ

These are on the same base CQ and between the same parallels CQ and AD. Therefore,

Area (ΔDCQ) = Area (ΔACQ)

□ Area (ΔDCQ) – Area (ΔPQC) = Area (ΔACQ) – Area (ΔPQC)

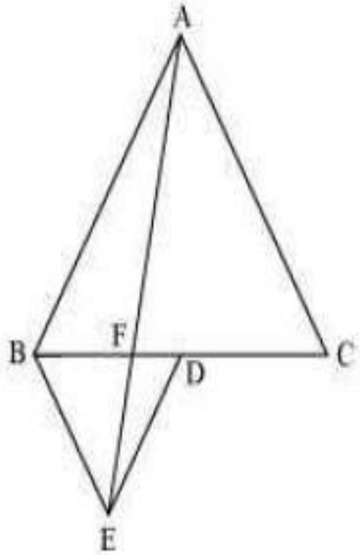
□ Area (ΔDPQ) = Area (ΔAPC) ... (2)

From equations (1) and (2), we obtain

Area (ΔBPC) = Area (ΔDPQ)

Question 5:

In the following figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that



(i) $ar(BDE) = \frac{1}{4} ar(ABC)$

(ii) $ar(BDE) = \frac{1}{2} ar(BAE)$

(iii) $ar(ABC) = 2 ar(BEC)$

(iv) $ar(BFE) = ar(AFD)$

(v) $ar(BFE) = 2 ar(FED)$

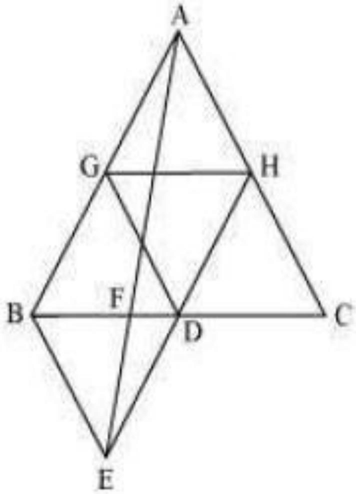
(vi) $ar(FED) = \frac{1}{8} ar(AFC)$

[Hint: Join EC and AD. Show that BE || AC and DE || AB, etc.]

Answer:

(i) Let G and H be the mid-points of side AB and AC respectively.

Line segment GH is joining the mid-points. Therefore, it will be parallel to third side BC and also its length will be half of the length of BC (mid-point theorem).



☐ GH = $\frac{1}{2}$ BC and GH || BD

☐ GH = BD = DC and GH || BD (D is the mid-point of BC)

Consider quadrilateral GHDB.

GH ||BD and GH = BD

Two line segments joining two parallel line segments of equal length will also be equal and parallel to each other.

Therefore, BG = DH and BG || DH

Hence, quadrilateral GHDB is a parallelogram.

We know that in a parallelogram, the diagonal bisects it into two triangles of equal area.

Hence, Area (ΔBDG) = Area (ΔHGD)

Similarly, it can be proved that quadrilaterals DCHG, GDHA, and BEDG are parallelograms and their respective diagonals are dividing them into two triangles of equal area.

ar (ΔGDH) = ar (ΔCHD) (For parallelogram DCHG)

ar (ΔGDH) = ar (ΔHAG) (For parallelogram GDHA)

ar (ΔBDE) = ar (ΔDBG) (For parallelogram BEDG)

ar (ΔABC) = ar(ΔBDG) + ar(ΔGDH) + ar(ΔDCH) + ar(ΔAGH)

ar (ΔABC) = 4 × ar(ΔBDE)

Hence, $\text{ar}(\text{BDE}) = \frac{1}{4}\text{ar}(\text{ABC})$

(ii)Area (ΔBDE) = Area (ΔAED) (Common base DE and DE||AB)

Area (ΔBDE) − Area (ΔFED) = Area (ΔAED) − Area (ΔFED)

Area (ΔBEF) = Area (ΔAFD) (1)

Area (ΔABD) = Area (ΔABF) + Area (ΔAFD)

Area (ΔABD) = Area (ΔABF) + Area (ΔBEF) [From equation (1)]

Area (ΔABD) = Area (ΔABE) (2)

AD is the median in ΔABC.

$$\text{ar}(\Delta\text{ABD}) = \frac{1}{2}\text{ar}(\Delta\text{ABC})$$

$$= \frac{4}{2}\text{ar}(\Delta\text{BDE}) \qquad \text{(As proved earlier)}$$

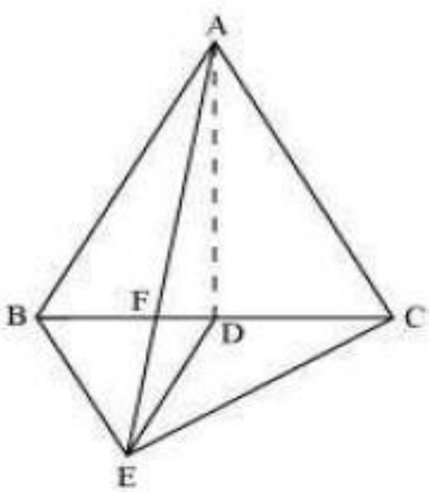
$$\text{ar}(\Delta\text{ABD}) = 2\,\text{ar}(\Delta\text{BDE}) \qquad (3)$$

From (2) and (3), we obtain

2 ar (ΔBDE) = ar (ΔABE)

Or, $\text{ar}(\Delta\text{BDE}) = \frac{1}{2}\text{ar}(\Delta\text{ABE})$

(iii)



ar (ΔABE) = ar (ΔBEC) (Common base BE and BE||AC)

ar (ΔABF) + ar (ΔBEF) = ar (ΔBEC)

Using equation (1), we obtain

ar (ΔABF) + ar (ΔAFD) = ar (ΔBEC)

ar (ΔABD) = ar (ΔBEC)

$$\frac{1}{2}\text{ar }(\Delta\text{ABC}) = \text{ar }(\Delta\text{BEC})$$

ar (ΔABC) = 2 ar (ΔBEC)

(iv)It is seen that ΔBDE and ar ΔAED lie on the same base (DE) and between the parallels DE and AB.

□ar (ΔBDE) = ar (ΔAED)

□ ar (ΔBDE) − ar (ΔFED) = ar (ΔAED) − ar (ΔFED)

□ar (ΔBFE) = ar (ΔAFD)

(v)Let *h* be the height of vertex E, corresponding to the side BD in ΔBDE.

Let *H* be the height of vertex A, corresponding to the side BC in ΔABC.

In (i), it was shown that $\text{ar}(\text{BDE}) = \frac{1}{4}\text{ar}(\text{ABC})$.

$$\therefore \frac{1}{2}\times \text{BD}\times h = \frac{1}{4}\left(\frac{1}{2}\times \text{BC}\times H\right)$$

$$\Rightarrow \text{BD}\times h = \frac{1}{4}(2\text{BD}\times H)$$

$$\Rightarrow h = \frac{1}{2}H$$

In (iv), it was shown that ar (ΔBFE) = ar (ΔAFD).

□ ar (ΔBFE) = ar (ΔAFD)

$$= \frac{1}{2}\times \text{FD}\times H = \frac{1}{2}\times \text{FD}\times 2h = 2\left(\frac{1}{2}\times \text{FD}\times h\right)$$

= 2 ar (ΔFED)

Hence, $\text{ar}(\text{BFE}) = 2\text{ar}(\text{FED})$.

(vi) Area (AFC) = area (AFD) + area (ADC)

$= ar(BFE) + \frac{1}{2} ar(ABC) \quad \left[\text{In (iv)}, ar(BFE) = ar(AFD)\ ;\ \text{AD is median of } \Delta ABC\right]$

$= ar(BFE) + \frac{1}{2} \times 4ar(BDE) \quad \left[\text{In (i)}, ar(BDE) = \frac{1}{4} ar(ABC)\right]$

$= ar(BFE) + 2ar(BDE) \quad \ldots(5)$

Now, by (v), $ar(BFE) = 2ar(FED)$. ... (6)

$ar(BDE) = ar(BFE) + ar(FED) = 2ar(FED) + ar(FED) = 3ar(FED) \quad \ldots(7)$

Therefore, from equations (5), (6), and (7), we get:

$ar(AFC) = 2ar(FED) + 2 \times 3ar(FED) = 8ar(FED)$

$\therefore ar(AFC) = 8ar(FED)$

Hence, $ar(FED) = \frac{1}{8} ar(AFC)$

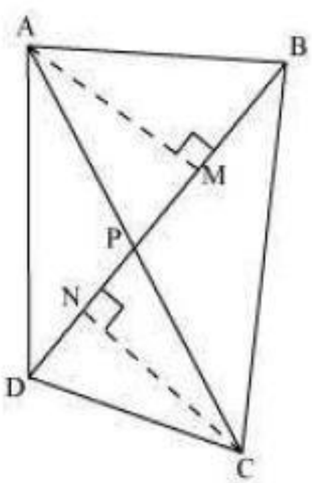
Question 6:

Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that $ar(APB) \times ar(CPD) = ar(APD) \times ar(BPC)$

[Hint: From A and C, draw perpendiculars to BD]

Answer:

Let us draw AM □ BD and CN □ BD



Area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$$ar(APB) \times ar(CPD) = \left[\frac{1}{2} \times BP \times AM\right] \times \left[\frac{1}{2} \times PD \times CN\right]$$

$$= \frac{1}{4} \times BP \times AM \times PD \times CN$$

$$ar(APD) \times ar(BPC) = \left[\frac{1}{2} \times PD \times AM\right] \times \left[\frac{1}{2} \times CN \times BP\right]$$

$$= \frac{1}{4} \times PD \times AM \times CN \times BP$$

$$= \frac{1}{4} \times BP \times AM \times PD \times CN$$

□ ar (APB) × ar (CPD) = ar (APD) × ar (BPC)

Question 7:

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

(i) $ar(PRQ) = \frac{1}{2} ar(ARC)$ (ii) $ar(RQC) = \frac{3}{8} ar(ABC)$

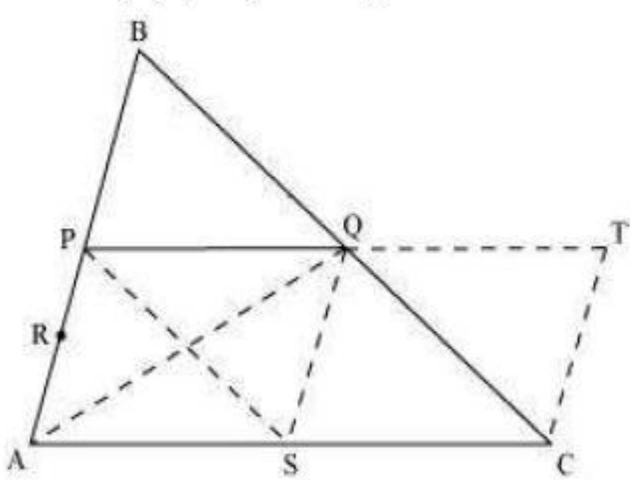
(iii) $ar(PBQ) = ar(ARC)$

Answer:

Take a point S on AC such that S is the mid-point of AC.

Extend PQ to T such that PQ = QT.

Join TC, QS, PS, and AQ.



In ΔABC, P and Q are the mid-points of AB and BC respectively. Hence, by using mid-point theorem, we obtain

PQ || AC and PQ $= \frac{1}{2}$AC

□ PQ || AS and PQ = AS (As S is the mid-point of AC)

□ PQSA is a parallelogram. We know that diagonals of a parallelogram bisect it into equal areas of triangles.

□ ar (ΔPAS) = ar (ΔSQP) = ar (ΔPAQ) = ar (ΔSQA)

Similarly, it can also be proved that quadrilaterals PSCQ, QSCT, and PSQB are also parallelograms and therefore,

ar (ΔPSQ) = ar (ΔCQS) (For parallelogram PSCQ)

ar (ΔQSC) = ar (ΔCTQ) (For parallelogram QSCT)

ar (ΔPSQ) = ar (ΔQBP) (For parallelogram PSQB)

Thus,

ar (ΔPAS) = ar (ΔSQP) = ar (ΔPAQ) = ar (ΔSQA) = ar (ΔQSC) = ar (ΔCTQ) = ar (ΔQBP) ... (1)

Also, ar (ΔABC) = ar (ΔPBQ) + ar (ΔPAS) + ar (ΔPQS) + ar (ΔQSC)

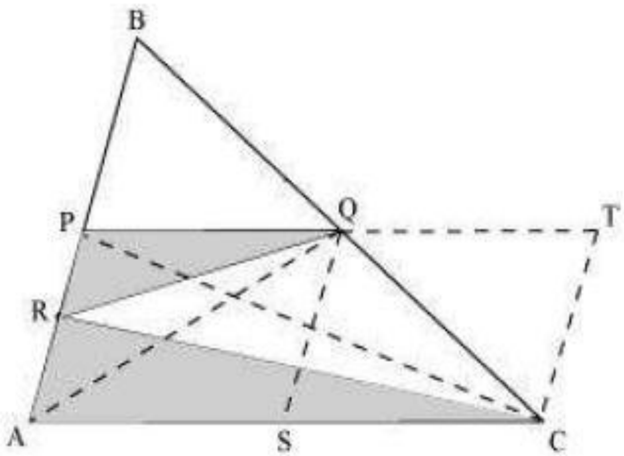
ar (ΔABC) = ar (ΔPBQ) + ar (ΔPBQ) + ar (ΔPBQ) + ar (ΔPBQ)

= ar (ΔPBQ) + ar (ΔPBQ) + ar (ΔPBQ) + ar (ΔPBQ)

= 4 ar (ΔPBQ)

□ ar (ΔPBQ) = $\frac{1}{4}$ ar (ΔABC) ... (2)

(i)Join point P to C.



In ΔPAQ, QR is the median.

$$\therefore ar(\Delta PRQ) = \frac{1}{2} ar(\Delta PAQ) = \frac{1}{2} \times \frac{1}{4} ar(\Delta ABC) = \frac{1}{8} ar(\Delta ABC)$$ ... (3)

In ΔABC, P and Q are the mid-points of AB and BC respectively. Hence, by using mid-point theorem, we obtain

PQ $= \frac{1}{2} AC$

$AC = 2PQ \Rightarrow AC = PT$

Also, PQ || AC $\Rightarrow$ PT || AC

Hence, PACT is a parallelogram.

ar (PACT) = ar (PACQ) + ar (ΔQTC)

= ar (PACQ) + ar (ΔPBQ [Using equation (1)]

☐ ar (PACT) = ar (ΔABC) ... (4)

$$\text{ar}(\Delta ARC) = \frac{1}{2}\text{ar}(\Delta PAC) \qquad (\text{CR is the median of } \Delta PAC)$$

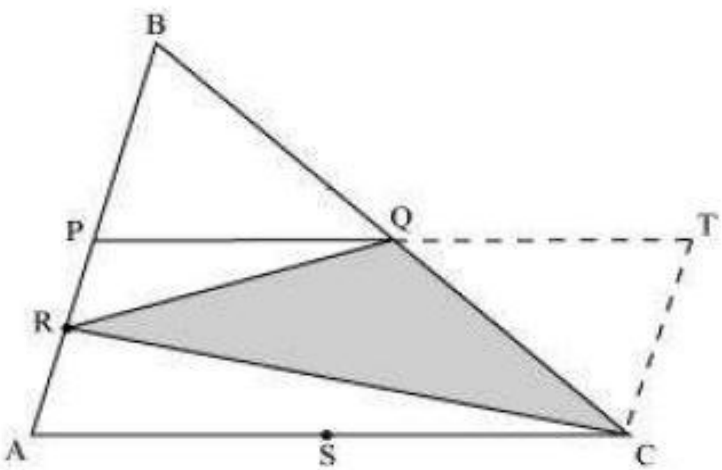
$$= \frac{1}{2} \times \frac{1}{2}\text{ar}(PACT) \; (\text{PC is the diagonal of parallelogram PACT})$$

$$= \frac{1}{4}\text{ar}(\Delta PACT) = \frac{1}{4}\text{ar}(\Delta ABC)$$

$$\Rightarrow \frac{1}{2}\text{ar}(\Delta ARC) = \frac{1}{8}\text{ar}(\Delta ABC)$$

$$\Rightarrow \frac{1}{2}\text{ar}(\Delta ARC) = \text{ar}(\Delta PRQ) \; [\text{Using equation (3)}] \quad \ldots (5)$$

(ii)



$$\text{ar}(PACT) = \text{ar}(\Delta PRQ) + \text{ar}(\Delta ARC) + \text{ar}(\Delta QTC) + \text{ar}(\Delta RQC)$$

Putting the values from equations (1), (2), (3), (4), and (5), we obtain

$$\text{ar}(\Delta ABC) = \frac{1}{8}\text{ar}(\Delta ABC) + \frac{1}{4}\text{ar}(\Delta ABC) + \frac{1}{4}\text{ar}(\Delta ABC) + \text{ar}(\Delta RQC)$$

$$\text{ar}(\Delta ABC) = \frac{5}{8}\text{ar}(\Delta ABC) + \text{ar}(\Delta RQC)$$

$$\text{ar}(\Delta RQC) = \left(1 - \frac{5}{8}\right)\text{ar}(\Delta ABC)$$

$$\text{ar}(\Delta RQC) = \frac{3}{8}\text{ar}(\Delta ABC)$$

(iii)In parallelogram PACT,

$$\text{ar}(\Delta ARC) = \frac{1}{2}\text{ar}(\Delta PAC) \quad (\text{CR is the median of } \Delta PAC)$$

$$= \frac{1}{2} \times \frac{1}{2}\text{ar}(PACT) \; (\text{PC is the diagonal of parallelogram PACT})$$

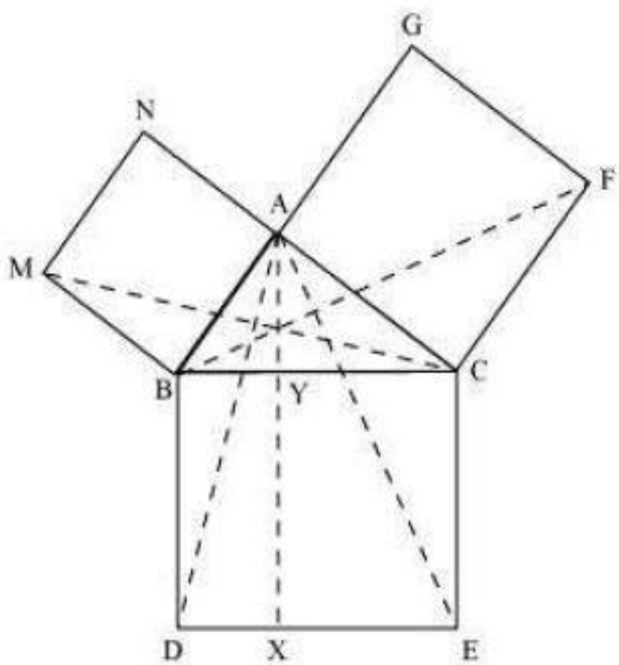
$$= \frac{1}{4}\text{ar}(\Delta PACT)$$

$$= \frac{1}{4}\text{ar}(\Delta ABC)$$

$$= \text{ar}(\Delta PBQ)$$

Question 8:

In the following figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ☐ DE meets BC at Y. Show that:



(i) ΔMBC ☐ ΔABD

(ii) $\text{ar}(BYXD) = 2\text{ar}(MBC)$

(iii) $\text{ar}(BYXD) = 2\text{ar}(ABMN)$

(iv) ΔFCB ☐ ΔACE

(v) $ar(CYXE) = 2ar(FCB)$

(vi) $ar(CYXE) = ar(ACFG)$

(vii) $ar(BCED) = ar(ABMN) + ar(ACFG)$

Note: Result (vii) is the famous *Theorem of Pythagoras*. You shall learn a simpler proof of this theorem in class X.

Answer:

(i) We know that each angle of a square is 90°.

Hence, □ABM = □DBC = 90º

□ □ABM + □ABC = □DBC + □ABC

□ □MBC = □ABD

In ΔMBC and ΔABD,

□MBC = □ABD (Proved above)

MB = AB (Sides of square ABMN)

BC = BD (Sides of square BCED)

□ ΔMBC □ ΔABD (SAS congruence rule)

(ii) We have

ΔMBC □ ΔABD

□ ar (ΔMBC) = ar (ΔABD) ... (1)

It is given that AX □ DE and BD □ DE (Adjacent sides of square BDEC)

□ BD || AX (Two lines perpendicular to same line are parallel to each other)

ΔABD and parallelogram BYXD are on the same base BD and between the same parallels BD and AX.

$\therefore ar\ (\Delta ABD) = \frac{1}{2} ar(BYXD)$

$ar\ (BYXD) = 2ar\ (\Delta ABD)$

Area (BYXD) = 2 area (ΔMBC) [Using equation (1)] ... (2)

(iii) ΔMBC and parallelogram ABMN are lying on the same base MB and between same parallels MB and NC.

$$\therefore \text{ar }(\Delta MBC) = \frac{1}{2}\text{ar}(ABMN)$$

2 ar (ΔMBC) = ar (ABMN)

ar (BYXD) = ar (ABMN) [Using equation (2)] ... (3)

(iv) We know that each angle of a square is 90°.

☐ ☐FCA = ☐BCE = 90°

☐ ☐FCA + ☐ACB = ☐BCE + ☐ACB

☐ ☐FCB = ☐ACE

In ΔFCB and ΔACE,

☐FCB = ☐ACE

FC = AC (Sides of square ACFG)

CB = CE (Sides of square BCED)

ΔFCB ☐ ΔACE (SAS congruence rule)

(v) It is given that AX ☐ DE and CE ☐ DE (Adjacent sides of square BDEC)

Hence, CE || AX (Two lines perpendicular to the same line are parallel to each other)

Consider ΔACE and parallelogram CYXE

ΔACE and parallelogram CYXE are on the same base CE and between the same parallels CE and AX.

$$\therefore \text{ar }(\Delta ACE) = \frac{1}{2}\text{ar }(CYXE)$$

☐ ar (CYXE) = 2 ar (ΔACE) ... (4)

We had proved that

☐ ΔFCB ☐ ΔACE

ar (ΔFCB) ☐ ar (ΔACE) ... (5)

On comparing equations (4) and (5), we obtain

ar (CYXE) = 2 ar (ΔFCB) ... (6)

(vi) Consider ΔFCB and parallelogram ACFG

ΔFCB and parallelogram ACFG are lying on the same base CF and between the same parallels CF and BG.

$\therefore \text{ar}(\Delta FCB) = \frac{1}{2}\text{ar}(ACFG)$

□ ar (ACFG) = 2 ar (ΔFCB)

□ ar (ACFG) = ar (CYXE) [Using equation (6)] ... (7)

(vii) From the figure, it is evident that

ar (BCED) = ar (BYXD) + ar (CYXE)

□ ar (BCED) = ar (ABMN) + ar (ACFG) [Using equations (3) and (7)]

NCERT

Class 9th Maths

Chapter 10: Circles

Exercise 10.1

Question 1:

Fill in the blanks

(i) The centre of a circle lies in ___________ of the circle. (exterior/ interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in ___________ of the circle. (exterior/ interior)

(iii) The longest chord of a circle is a ___________ of the circle.

(iv) An arc is a ___________ when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and ___________ of the circle.

(vi) A circle divides the plane, on which it lies, in ___________ parts.

Answer:

(i) The centre of a circle lies in interior of the circle.

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in exterior of the circle.

(iii) The longest chord of a circle is a diameter of the circle.

(iv) An arc is a semi-circle when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and chord of the circle.

(vi) A circle divides the plane, on which it lies, in three parts.

Question 2:

Write True or False: Give reasons for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

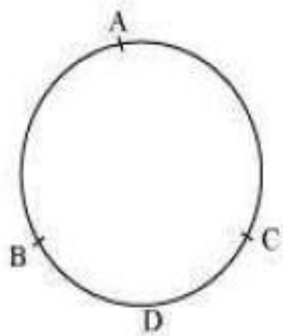
(vi) A circle is a plane figure.

Answer:

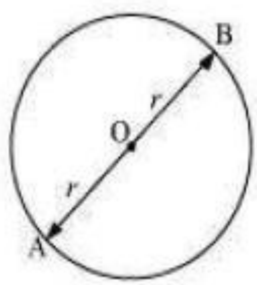
(i) True. All the points on the circle are at equal distances from the centre of the circle, and this equal distance is called as radius of the circle.

(ii) False. There are infinite points on a circle. Therefore, we can draw infinite number of chords of given length. Hence, a circle has infinite number of equal chords.

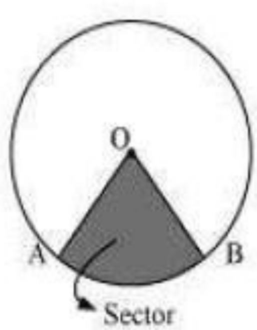
(iii) False. Consider three arcs of same length as AB, BC, and CA. It can be observed that for minor arc BDC, CAB is a major arc. Therefore, AB, BC, and CA are minor arcs of the circle.



(iv) True. Let AB be a chord which is twice as long as its radius. It can be observed that in this situation, our chord will be passing through the centre of the circle. Therefore, it will be the diameter of the circle.



(v) False. Sector is the region between an arc and two radii joining the centre to the end points of the arc. For example, in the given figure, OAB is the sector of the circle.



(vi) True. A circle is a two-dimensional figure and it can also be referred to as a plane figure.

Exercise 10.2

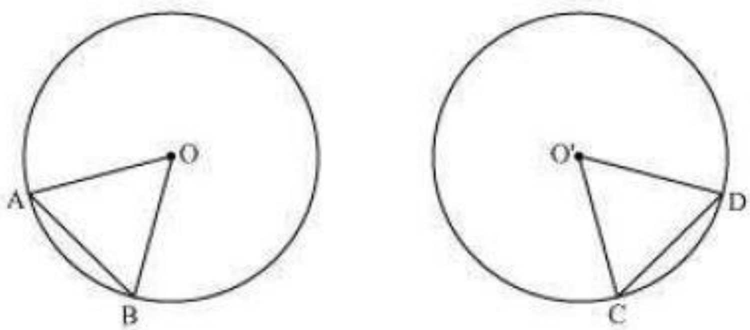
Question 1:

Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Answer:

A circle is a collection of points which are equidistant from a fixed point. This fixed point is called as the centre of the circle and this equal distance is called as radius of the circle. And thus, the shape of a circle depends on its radius. Therefore, it can be observed that if we try to superimpose two circles of equal radius, then both circles will cover each other. Therefore, two circles are congruent if they have equal radius.

Consider two congruent circles having centre O and O' and two chords AB and CD of equal lengths.



In ΔAOB and $\Delta CO'D$,

AB = CD (Chords of same length)

OA = O'C (Radii of congruent circles)

OB = O'D (Radii of congruent circles)

$\therefore \Delta AOB \cong \Delta CO'D$ (SSS congruence rule)

$\Rightarrow \angle AOB = \angle CO'D$ (By CPCT)

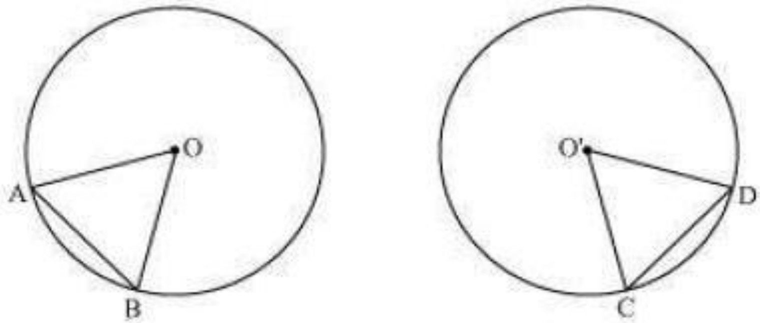
Hence, equal chords of congruent circles subtend equal angles at their centres.

Question 2:

Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Answer:

Let us consider two congruent circles (circles of same radius) with centres as O and O'.



In ΔAOB and ΔCO'D,

∠AOB = ∠CO'D (Given)

OA = O'C (Radii of congruent circles)

OB = O'D (Radii of congruent circles)

∴ ΔAOB ≅ ΔCO'D (SSS congruence rule)

⇒ AB = CD (By CPCT)

Hence, if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

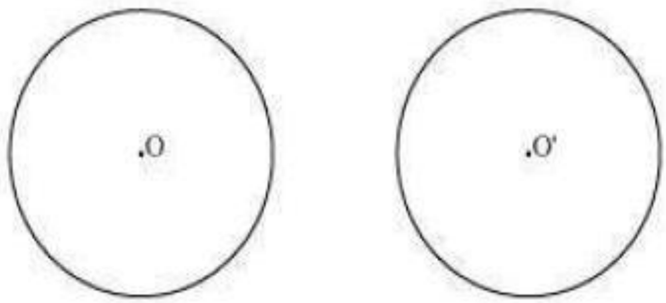
Exercise 10.3

Question 1:

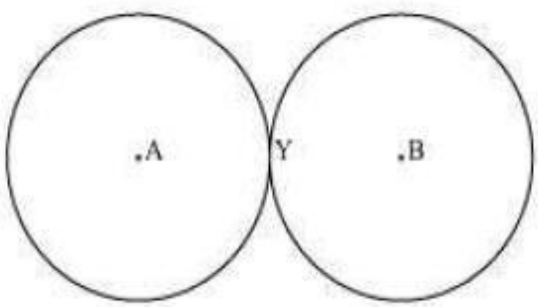
Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Answer:

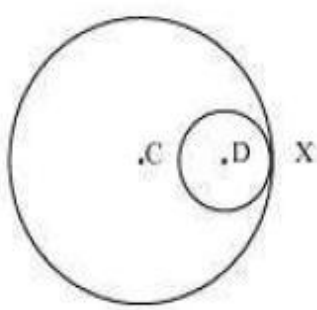
Consider the following pair of circles.



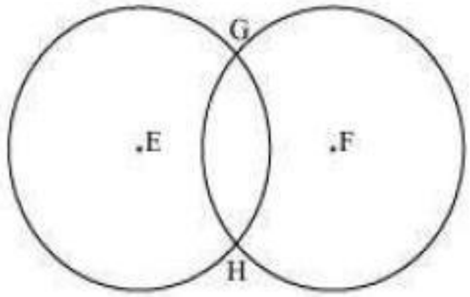
The above circles do not intersect each other at any point. Therefore, they do not have any point in common.



The above circles touch each other only at one point Y. Therefore, there is 1 point in common.



The above circles touch each other at 1 point X only. Therefore, the circles have 1 point in common.



These circles intersect each other at two points G and H. Therefore, the circles have two points in common. It can be observed that there can be a maximum of 2 points in common. Consider the situation in which two congruent circles are superimposed on each other. This situation can be referred to as if we are drawing the circle two times.

Question 2:

Suppose you are given a circle. Give a construction to find its centre.

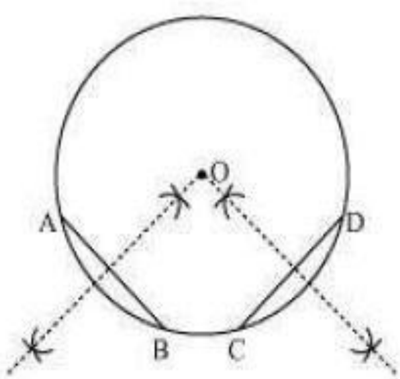
Answer:

The below given steps will be followed to find the centre of the given circle.

Step1. Take the given circle.

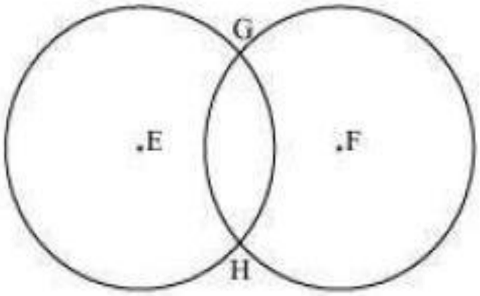
Step2. Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these chords.

Step3. Let these perpendicular bisectors meet at point O. Hence, O is the centre of the given circle.



Question 3:

If two circles intersect at two points, then prove that their centres lie on the perpendicular bisector of the common chord.



These circles intersect each other at two points G and H. Therefore, the circles have two points in common. It can be observed that there can be a maximum of 2 points in common. Consider the situation in which two congruent circles are superimposed on each other. This situation can be referred to as if we are drawing the circle two times.

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Step2. Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these chords.

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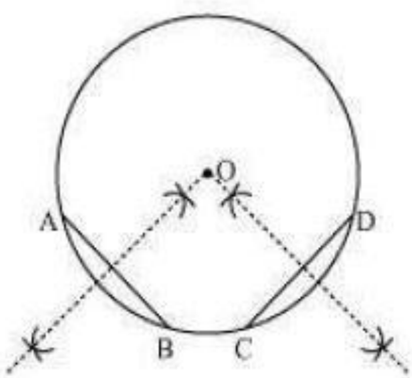

Question 3:

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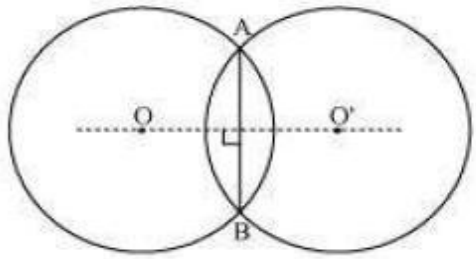


Answer:

Consider two circles centered at point O and O′, intersecting each other at point A and B respectively.

Join AB. AB is the chord of the circle centered at O. Therefore, perpendicular bisector of AB will pass through O.

Again, AB is also the chord of the circle centered at O′. Therefore, perpendicular bisector of AB will also pass through O′.

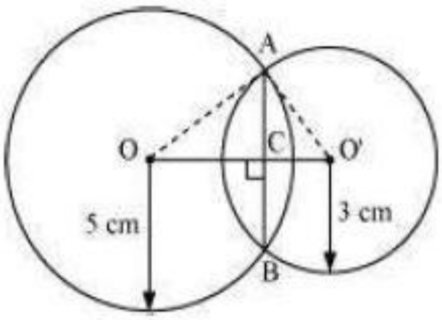
Clearly, the centres of these circles lie on the perpendicular bisector of the common chord.

Exercise 10.4

Question 1:

Two circles of radii 5 cm and 3 cm intersect at two points and the distance betwe their centres is 4 cm. Find the length of the common chord.

Answer:



Let the radius of the circle centered at O and O' be 5 cm and 3 cm respectively.

OA = OB = 5 cm

O'A = O'B = 3 cm

OO' will be the perpendicular bisector of chord AB.

∴ AC = CB

It is given that, OO' = 4 cm

Let OC be x. Therefore, O'C will be $4 - x$.

In ΔOAC,

$OA^2 = AC^2 + OC^2$

$\Rightarrow 5^2 = AC^2 + x^2$

$\Rightarrow 25 - x^2 = AC^2$ ... (1)

In ΔO'AC,

$O'A^2 = AC^2 + O'C^2$

$\Rightarrow 3^2 = AC^2 + (4 - x)^2$

$\Rightarrow 9 = AC^2 + 16 + x^2 - 8x$

$\Rightarrow AC^2 = - x^2 - 7 + 8x$ ... (2)

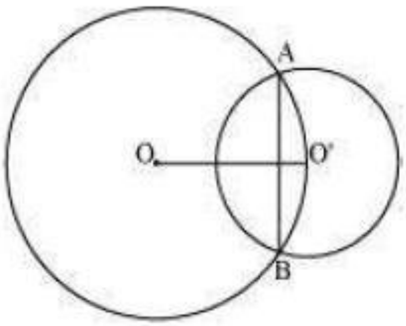
From equations (1) and (2), we obtain

$25 - x^2 = - x^2 - 7 + 8x$

$8x = 32$

$x = 4$

Therefore, the common chord will pass through the centre of the smaller circle i.e., O' and hence, it will be the diameter of the smaller circle.



$AC^2 = 25 - x^2 = 25 - 4^2 = 25 - 16 = 9$

$\therefore$ AC = 3 m

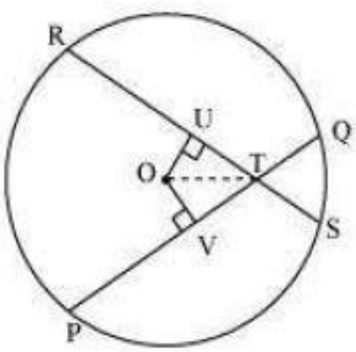
Length of the common chord AB = 2 AC = (2 × 3) m = 6 m

Question 2:

If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Answer:

Let PQ and RS be two equal chords of a given circle and they are intersecting each other at point T.



Draw perpendiculars OV and OU on these chords.

In ΔOVT and ΔOUT,

OV = OU (Equal chords of a circle are equidistant from the centre)

∠OVT = ∠OUT (Each 90°)

OT = OT (Common)

$\therefore$ ΔOVT ≅ ΔOUT (RHS congruence rule)

∴ VT = UT (By CPCT) … (1)

It is given that,

PQ = RS … (2)

$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}RS$

⇒ PV = RU … (3)

On adding equations (1) and (3), we obtain

PV + VT = RU + UT

⇒ PT = RT … (4)

On subtracting equation (4) from equation (2), we obtain

PQ − PT = RS − RT

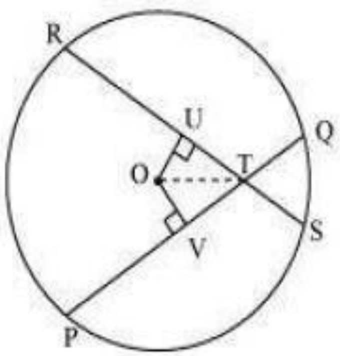
⇒ QT = ST … (5)

Equations (4) and (5) indicate that the corresponding segments of chords PQ and RS are congruent to each other.

Question 3:

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Answer:



Let PQ and RS are two equal chords of a given circle and they are intersecting each other at point T.

Draw perpendiculars OV and OU on these chords.

In ΔOVT and ΔOUT,

OV = OU (Equal chords of a circle are equidistant from the centre)

∠OVT = ∠OUT (Each 90°)

OT = OT (Common)

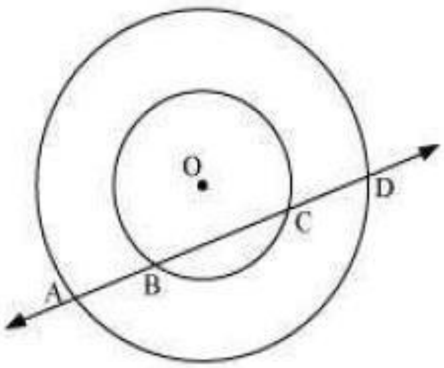
∴ ΔOVT ≅ ΔOUT (RHS congruence rule)

∴ ☐OTV = ☐OTU (By CPCT)

Therefore, it is proved that the line joining the point of intersection to the centre makes equal angles with the chords.

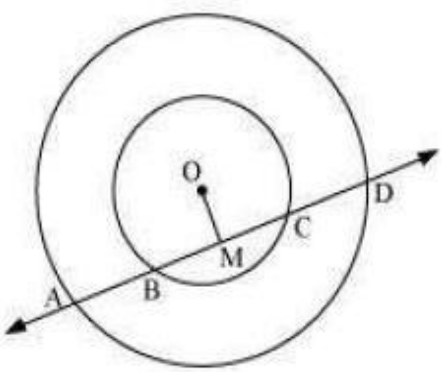
Question 4:

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure 10.25).



Answer:

Let us draw a perpendicular OM on line AD.



It can be observed that BC is the chord of the smaller circle and AD is the chord of the bigger circle.

We know that perpendicular drawn from the centre of the circle bisects the chord.

☐ BM = MC ... (1)

And, AM = MD ... (2)

On subtracting equation (2) from (1), we obtain

AM − BM = MD − MC

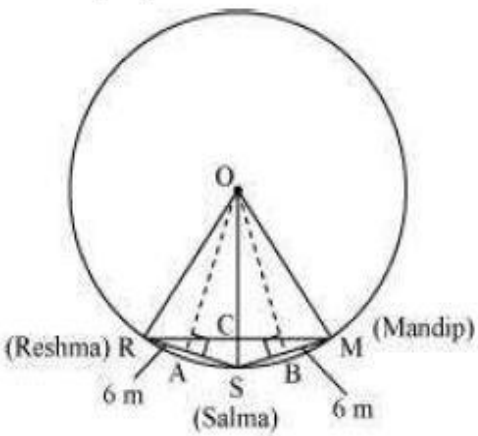
☐ AB = CD

Question 5:

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Answer:

Draw perpendiculars OA and OB on RS and SM respectively.



$$AR = AS = \frac{6}{2} = 3\,\text{m}$$

OR = OS = OM = 5 m. (Radii of the circle)

In ΔOAR,

$OA^2 + AR^2 = OR^2$

$OA^2 + (3\text{ m})^2 = (5\text{ m})^2$

$OA^2 = (25 - 9)\text{ m}^2 = 16\text{ m}^2$

OA = 4 m

ORSM will be a kite (OR = OM and RS = SM). We know that the diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangles is bisected by another diagonal.

☐☐RCS will be of 90° and RC = CM

\`

Area of ΔORS = $\frac{1}{2}\times OA\times RS$

$$\frac{1}{2}\times RC\times OS=\frac{1}{2}\times 4\times 6$$

$$RC\times 5=24$$

$$RC=4.8$$

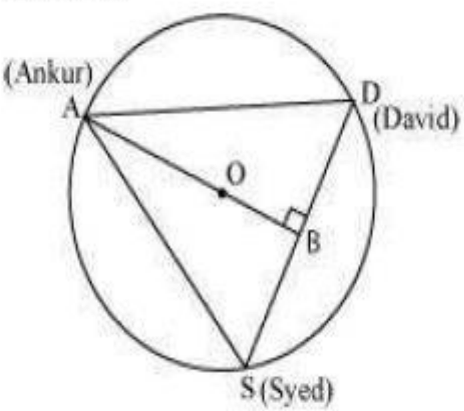
$$RM=2RC=2(4.8)=9.6$$

Therefore, the distance between Reshma and Mandip is 9.6 m.

Question 6:

A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Answer:



It is given that AS = SD = DA

Therefore, ΔASD is an equilateral triangle.

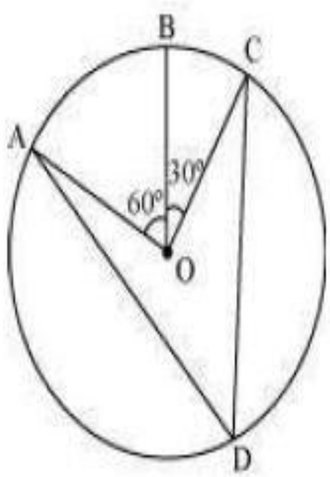
OA (radius) = 20 m

Medians of equilateral triangle pass through the circum centre (O) of the equilateral triangle ASD. We also know that medians intersect each other in the ratio 2: 1. As AB is the median of equilateral triangle ASD, we can write

Exercise 10.5

Question 1:

In the given figure, A, B and C are three points on a circle with centre O such that □BOC = 30° and □AOB = 60°. If D is a point on the circle other than the arc ABC, find □ADC.



Answer:

It can be observed that

□AOC = □AOB + □BOC

= 60° + 30°

= 90°

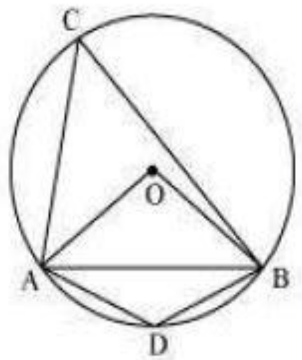
We know that angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$$\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2}\times 90° = 45°$$

Question 2:

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Answer:



In ΔOAB,

AB = OA = OB = radius

□ ΔOAB is an equilateral triangle.

Therefore, each interior angle of this triangle will be of 60°.

□ □AOB = 60°

$$\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2}(60°) = 30°$$

In cyclic quadrilateral ACBD,

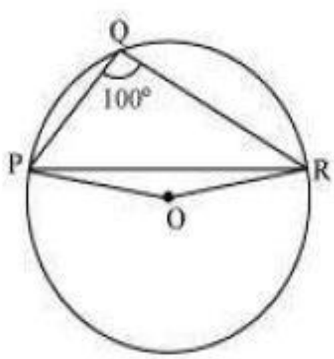
□ACB + □ADB = 180° (Opposite angle in cyclic quadrilateral)

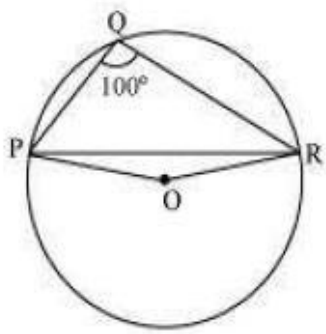
□ □ADB = 180° − 30° = 150°

Therefore, angle subtended by this chord at a point on the major arc and the minor arc are 30° and 150° respectively.

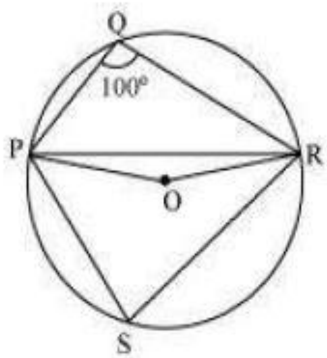
Question 3:

In the given figure, □PQR = 100°, where P, Q and R are points on a circle with centre O. Find □OPR.





Answer:



Consider PR as a chord of the circle.

Take any point S on the major arc of the circle.

PQRS is a cyclic quadrilateral.

□PQR + □PSR = 180° (Opposite angles of a cyclic quadrilateral)

□ □PSR = 180° − 100° = 80°

We know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

□ □POR = 2□PSR = 2 (80°) = 160°

In ΔPOR,

OP = OR (Radii of the same circle)

□ □OPR = □ORP (Angles opposite to equal sides of a triangle)

□OPR + □ORP + □POR = 180° (Angle sum property of a triangle)

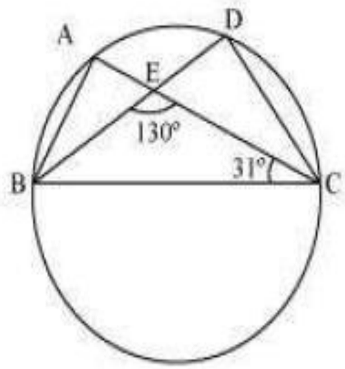
2 □OPR + 160° = 180°

2 □OPR = 180° − 160° = 20°

□OPR = 10°

Question 5:

In the given figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that □BEC = 130° and □ECD = 20°. Find □BAC.



Answer:

In ΔCDE,

□CDE + □DCE = □CEB (Exterior angle)

□ □CDE + 20° = 130°

□ □CDE = 110°

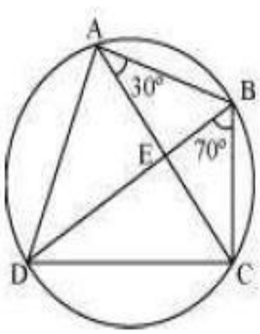
However, □BAC = □CDE (Angles in the same segment of a circle)

□ □BAC = 110°

Question 6:

ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If □DBC = 70°, □BAC is 30°, find □BCD. Further, if AB = BC, find □ECD.

Answer:



For chord CD,

□CBD = □CAD (Angles in the same segment)

□CAD = 70°

□BAD = □BAC + □CAD = 30° + 70° = 100°

□BCD + □BAD = 180° (Opposite angles of a cyclic quadrilateral)

□BCD + 100° = 180°

□BCD = 80°

In ΔABC,

AB = BC (Given)

□ □BCA = □CAB (Angles opposite to equal sides of a triangle)

□ □BCA = 30°

We have, □BCD = 80°

□ □BCA + □ACD = 80°

30° + □ACD = 80°

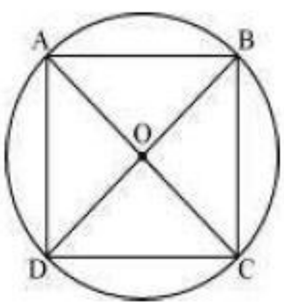
□ □ACD = 50°

□ □ECD = 50°

Question 7:

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Answer:



Let ABCD be a cyclic quadrilateral having diagonals BD and AC, intersecting each other at point O.

$\angle BAD = \frac{1}{2}\angle BOD = \frac{180^\circ}{2} = 90^\circ$ (Consider BD as a chord)

□BCD + □BAD = 180° (Cyclic quadrilateral)

□BCD = 180° − 90° = 90°

$\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2}(180^\circ) = 90^\circ$ (Considering AC as a chord)

□ADC + □ABC = 180° (Cyclic quadrilateral)

90° + □ABC = 180°

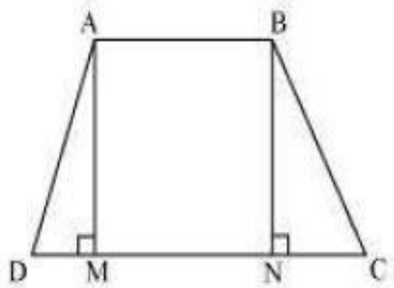
□ABC = 90°

Each interior angle of a cyclic quadrilateral is of 90°. Hence, it is a rectangle.

Question 8:

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Answer:



Consider a trapezium ABCD with AB | |CD and BC = AD.

Draw AM □ CD and BN □ CD.

In ΔAMD and ΔBNC,

AD = BC (Given)

□AMD = □BNC (By construction, each is 90°)

AM = BM (Perpendicular distance between two parallel lines is same)

□ ΔAMD □ ΔBNC (RHS congruence rule)

□ □ADC = □BCD (CPCT) ... (1)

□BAD and □ADC are on the same side of transversal AD.

□BAD + □ADC = 180° ... (2)

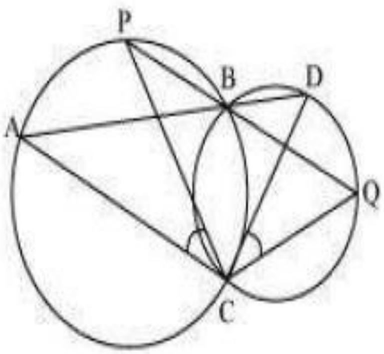
□BAD + □BCD = 180° [Using equation (1)]

This equation shows that the opposite angles are supplementary.

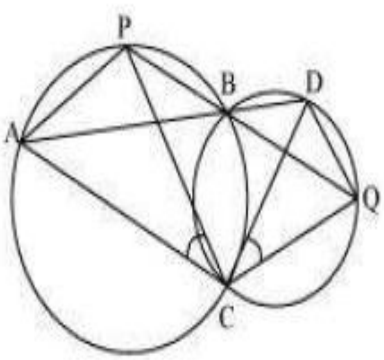
Therefore, ABCD is a cyclic quadrilateral.

Question 9:

Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see the given figure). Prove that □ACP = □QCD.



Answer:



Join chords AP and DQ.

For chord AP,

□PBA = □ACP (Angles in the same segment) ... (1)

For chord DQ,

□DBQ = □QCD (Angles in the same segment) ... (2)

ABD and PBQ are line segments intersecting at B.

□ □PBA = □DBQ (Vertically opposite angles) ... (3)

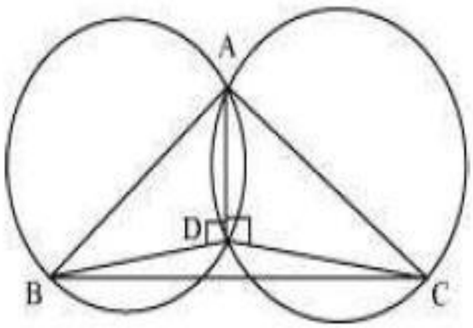
From equations (1), (2), and (3), we obtain

□ACP = □QCD

Question 10:

If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Answer:



Consider a ΔABC.

Two circles are drawn while taking AB and AC as the diameter.

Let they intersect each other at D and let D not lie on BC.

Join AD.

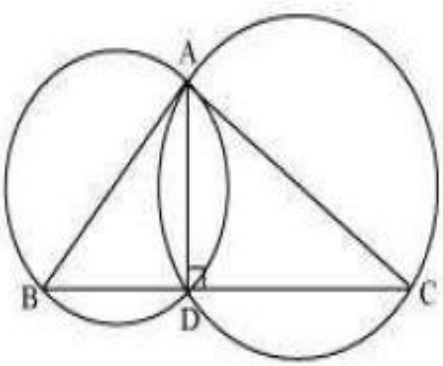
□ADB = 90° (Angle subtended by semi-circle)

□ADC = 90° (Angle subtended by semi-circle)

□BDC = □ADB + □ADC = 90° + 90° = 180°

Therefore, BDC is a straight line and hence, our assumption was wrong.

Thus, Point D lies on third side BC of ΔABC.

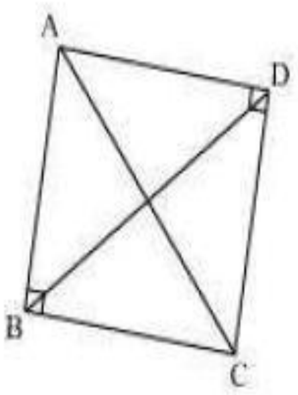


Question 11:

ABC and ADC are two right triangles with common hypotenuse AC. Prove that □CAD = □CBD.

Answer:

\



In ΔABC,

□ABC + □BCA + □CAB = 180° (Angle sum property of a triangle)

□ 90° + □BCA + □CAB = 180°

□ □BCA + □CAB = 90° ... (1)

In ΔADC,

□CDA + □ACD + □DAC = 180° (Angle sum property of a triangle)

□ 90° + □ACD + □DAC = 180°

□ □ACD + □DAC = 90° ... (2)

Adding equations (1) and (2), we obtain

□BCA + □CAB + □ACD + □DAC = 180°

□ (□BCA + □ACD) + (□CAB + □DAC) = 180°

□BCD + □DAB = 180° ... (3)

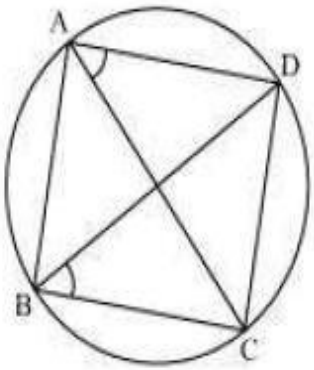
However, it is given that

□B + □D = 90° + 90° = 180° ... (4)

From equations (3) and (4), it can be observed that the sum of the measures of opposite angles of quadrilateral ABCD is 180°. Therefore, it is a cyclic quadrilateral.

Consider chord CD.

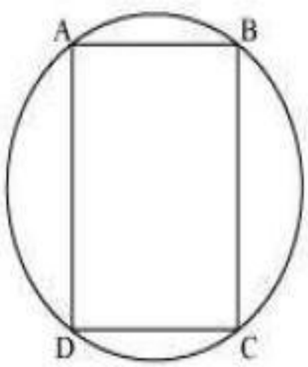
□CAD = □CBD (Angles in the same segment)



Question 12:

Prove that a cyclic parallelogram is a rectangle.

Answer:



Let ABCD be a cyclic parallelogram.

□A + □C = 180° (Opposite angles of a cyclic quadrilateral) ... (1)

We know that opposite angles of a parallelogram are equal.

□ □A = □C and □B = □D

From equation (1),

□A + □C = 180°

□ □A + □A = 180°

□ 2 □A = 180°

□ □A = 90°

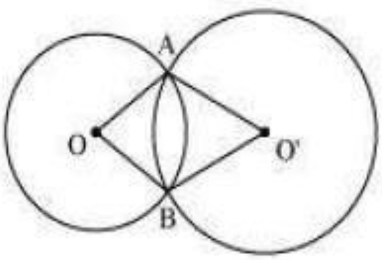
Parallelogram ABCD has one of its interior angles as 90°. Therefore, it is a rectangle.

Exercise 10.6

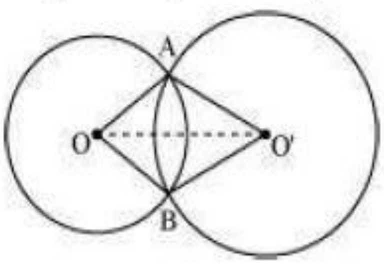
Question 1:

Prove that line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Answer:



Let two circles having their centres as O and O' intersect each other at point A and B respectively. Let us join OO'.



In $\Delta AOO'$ and BOO',

OA = OB (Radius of circle 1)

$O'A = O'B$ (Radius of circle 2)

$OO' = OO'$ (Common)

$\Delta AOO'$ □ $\Delta BOO'$ (By SSS congruence rule)

□OAO' = □OBO' (By CPCT)

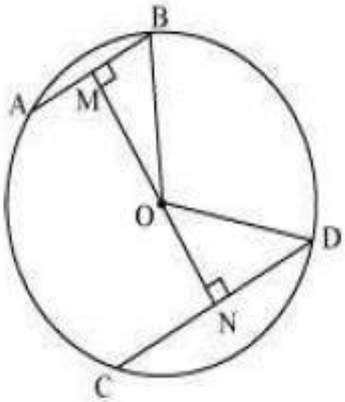
Therefore, line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Question 2:

Two chords AB and CD of lengths 5 cm 11cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Answer:

Draw OM □ AB and ON □ CD. Join OB and OD.



$$BM = \frac{AB}{2} = \frac{5}{2}$$ (Perpendicular from the centre bisects the chord)

$$ND = \frac{CD}{2} = \frac{11}{2}$$

Let ON be *x*. Therefore, OM will be 6− *x*.

In ΔMOB,

$$OM^2 + MB^2 = OB^2$$

$$(6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$$

$$36 + x^2 - 12x + \frac{25}{4} = OB^2 \qquad \dots (1)$$

In ΔNOD,

$$ON^2 + ND^2 = OD^2$$

$$x^2 + \left(\frac{11}{2}\right)^2 = OD^2$$

$$x^2 + \frac{121}{4} = OD^2 \qquad \dots (2)$$

We have OB = OD (Radii of the same circle)

Therefore, from equation (1) and (2),

$$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$= \frac{144 + 25 - 121}{4} \quad = \frac{48}{4} = 12$$

$$x = 1$$

From equation (2),

$$(1)^2 + \left(\frac{121}{4}\right) = OD^2$$

$$OD^2 = 1 + \frac{121}{4} = \frac{125}{4}$$

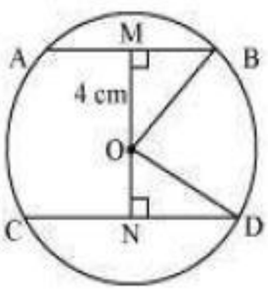
$$OD = \frac{5}{2}\sqrt{5}$$

Therefore, the radius of the circle is $\frac{5}{2}\sqrt{5}$ cm.

Question 3:

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Answer:



Let AB and CD be two parallel chords in a circle centered at O. Join OB and OD.

Distance of smaller chord AB from the centre of the circle = 4 cm

OM = 4 cm

MB = $\frac{AB}{2} = \frac{6}{2} = 3\text{cm}$

In ΔOMB,

$OM^2 + MB^2 = OB^2$

$(4)^2 + (3)^2 = OB^2$

$16 + 9 = OB^2$

$OB = \sqrt{25}$

$OB = 5\,cm$

In ΔOND,

$OD = OB = 5\,cm$ (Radii of the same circle)

$ND = \frac{CD}{2} = \frac{8}{2} = 4cm$

$ON^2 + ND^2 = OD^2$

$ON^2 + (4)^2 = (5)^2$

$ON^2 = 25 - 16 = 9$

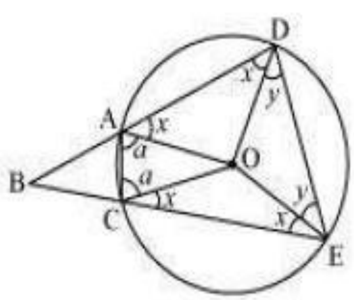
$ON = 3$

Therefore, the distance of the bigger chord from the centre is 3 cm.

Question 4:

Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that □ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Answer:



In ΔAOD and ΔCOE,

OA = OC (Radii of the same circle)

OD = OE (Radii of the same circle)

AD = CE (Given)

☐ ΔAOD ☐ ΔCOE (SSS congruence rule)

☐OAD = ☐OCE (By CPCT) ... (1)

☐ODA = ☐OEC (By CPCT) ... (2)

Also,

☐OAD = ☐ODA (As OA = OD) ... (3)

From equations (1), (2), and (3), we obtain

☐OAD = ☐OCE = ☐ODA = ☐OEC

Let ☐OAD = ☐OCE = ☐ODA = ☐OEC = x

In Δ OAC,

OA = OC

☐ ☐OCA = ☐OAC (Let a)

In Δ ODE,

OD = OE

☐OED = ☐ODE (Let y)

ADEC is a cyclic quadrilateral.

☐ ☐CAD + ☐DEC = 180º (Opposite angles are supplementary)

$x + a + x + y = 180^\circ$

$2x + a + y = 180^\circ$

$y = 180^\circ - 2x - a$ ... (4)

However, ☐DOE = 180º − $2y$

And, ☐AOC = 180º − $2a$

☐DOE − ☐AOC = $2a - 2y = 2a - 2(180^\circ - 2x - a)$

$= 4a + 4x - 360^\circ$ ... (5)

☐BAC + ☐CAD = 180º (Linear pair)

☐ ☐BAC = 180º − ☐CAD = 180º − $(a + x)$

Similarly, ☐ACB = 180º − $(a + x)$

In ΔABC,

☐ABC + ☐BAC + ☐ACB = 180º (Angle sum property of a triangle)

☐ABC = 180º − ☐BAC − ☐ACB

$= 180^\circ - (180^\circ - a - x) - (180^\circ - a - x)$

$= 2a + 2x - 180^\circ$

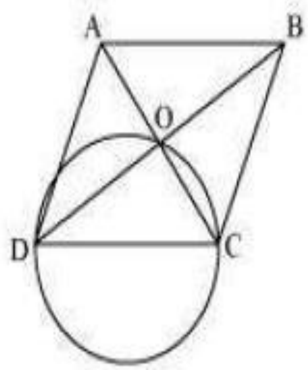
$= \frac{1}{2}[4a + 4x - 360^\circ]$

□ABC = $\frac{1}{2}$[□DOE – □ AOC] [Using equation (5)]

Question 5:

Prove that the circle drawn with any side of a rhombus as diameter passes through the point of intersection of its diagonals.

Answer:



Let ABCD be a rhombus in which diagonals are intersecting at point O and a circle is drawn while taking side CD as its diameter. We know that a diameter subtends 90° on the arc.

□ □COD = 90°

Also, in rhombus, the diagonals intersect each other at 90°.

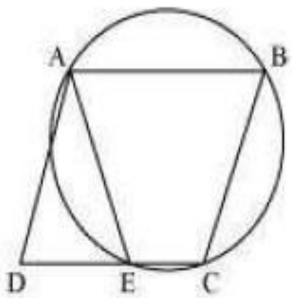
□AOB = □BOC = □COD = □DOA = 90°

Clearly, point O has to lie on the circle.

Question 6:

ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

Answer:



It can be observed that ABCE is a cyclic quadrilateral and in a cyclic quadrilateral, the sum of the opposite angles is 180°.

☐AEC + ☐CBA = 180°

☐AEC + ☐AED = 180° (Linear pair)

☐AED = ☐CBA … (1)

For a parallelogram, opposite angles are equal.

☐ADE = ☐CBA … (2)

From (1) and (2),

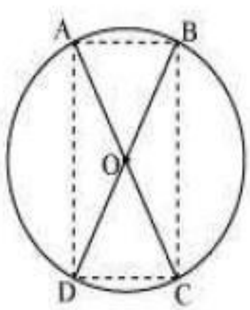
☐AED = ☐ADE

AD = AE (Angles opposite to equal sides of a triangle)

Question 7:

AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.

Answer:



Let two chords AB and CD are intersecting each other at point O.

In ΔAOB and ΔCOD,

OA = OC (Given)

OB = OD (Given)

□AOB = □COD (Vertically opposite angles)

ΔAOB □ ΔCOD (SAS congruence rule)

AB = CD (By CPCT)

Similarly, it can be proved that ΔAOD □ ΔCOB

□ AD = CB (By CPCT)

Since in quadrilateral ACBD, opposite sides are equal in length, ACBD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

□ □A = □C

However, □A + □C = 180° (ABCD is a cyclic quadrilateral)

□ □A + □A = 180°

□ 2 □A = 180°

□ □A = 90°

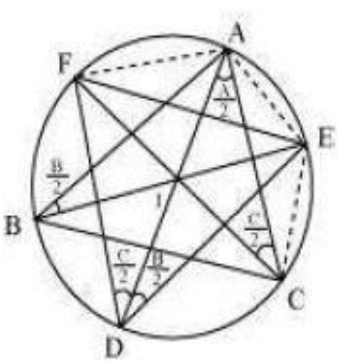
As ACBD is a parallelogram and one of its interior angles is 90°, therefore, it is a rectangle.

□A is the angle subtended by chord BD. And as □A = 90°, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

Question 8:

Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are 90° $-\frac{1}{2}A,\ 90^\circ-\frac{1}{2}B \text{ and } 90^\circ-\frac{1}{2}C$.

Answer:



It is given that BE is the bisector of □B.

☐ ☐ABE = $\frac{\angle B}{2}$

However, ☐ADE = ☐ABE (Angles in the same segment for chord AE)

☐ ☐ADE = $\frac{\angle B}{2}$

Similarly, ☐ACF = ☐ADF = $\frac{\angle C}{2}$ (Angle in the same segment for chord AF)

☐D = ☐ADE + ☐ADF

$=\frac{\angle B}{2}+\frac{\angle C}{2}$

$=\frac{1}{2}(\angle B+\angle C)$

$=\frac{1}{2}(180^\circ-\angle A)$

$=90^\circ-\frac{1}{2}\angle A$

Similarly, it can be proved that

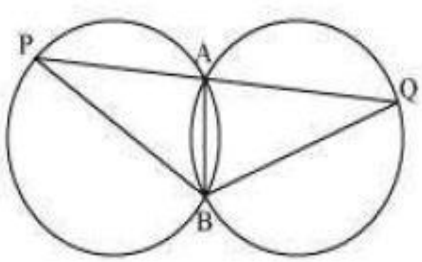
$\angle E=90^\circ-\frac{1}{2}\angle B$

$\angle F=90^\circ-\frac{1}{2}\angle C$

Question 9:

Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Answer:



AB is the common chord in both the congruent circles.

□ □APB = □AQB

In ΔBPQ,

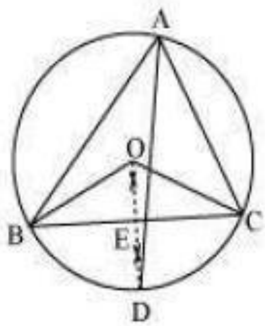
□APB = □AQB

□ BQ = BP (Angles opposite to equal sides of a triangle)

Question 10:

In any triangle ABC, if the angle bisector of □A and perpendicular bisector of BC intersect, prove that they intersect on the circum circle of the triangle ABC.

Answer:



Let perpendicular bisector of side BC and angle bisector of □A meet at point D. Let the perpendicular bisector of side BC intersect it at E.

Perpendicular bisector of side BC will pass through circumcentre O of the circle. □BOC and □BAC are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively. We also know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

□BOC = 2 □BAC = 2 □A ... (1)

In ΔBOE and ΔCOE,

OE = OE (Common)

OB = OC (Radii of same circle)

□OEB = □OEC (Each 90° as OD □ BC)

□ ΔBOE □ □COE (RHS congruence rule)

□BOE = □COE (By CPCT) ... (2)

However, □BOE + □COE = □BOC

□ □BOE +□BOE = 2 □A [Using equations (1) and (2)]

`

□ 2 □BOE = 2 □A

□ □BOE = □A

□ □BOE = □COE = □A

The perpendicular bisector of side BC and angle bisector of □A meet at point D.

□ □BOD = □BOE = □A ... (3)

Since AD is the bisector of angle □A,

□BAD = $\frac{\angle A}{2}$

□ 2 □BAD = □A ... (4)

From equations (3) and (4), we obtain

□BOD = 2 □BAD

This can be possible only when point BD will be a chord of the circle. For this, the point D lies on the circum circle.

Therefore, the perpendicular bisector of side BC and the angle bisector of □A meet on the circum circle of triangle ABC.

NCERT
Class 9th Maths
Chapter 11: Constructions

Exercise 11.1

Question 1:

Construct an angle of 90° at the initial point of a given ray and justify the construction.

Answer:

The below given steps will be followed to construct an angle of 90°.

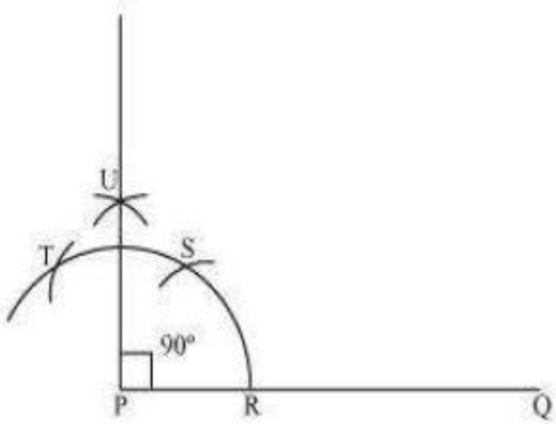
(i) Take the given ray PQ. Draw an arc of some radius taking point P as its centre, which intersects PQ at R.

(ii) Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at S.

(iii) Taking S as centre and with the same radius as before, draw an arc intersecting the arc at T (see figure).

(iv) Taking S and T as centre, draw an arc of same radius to intersect each other at U.

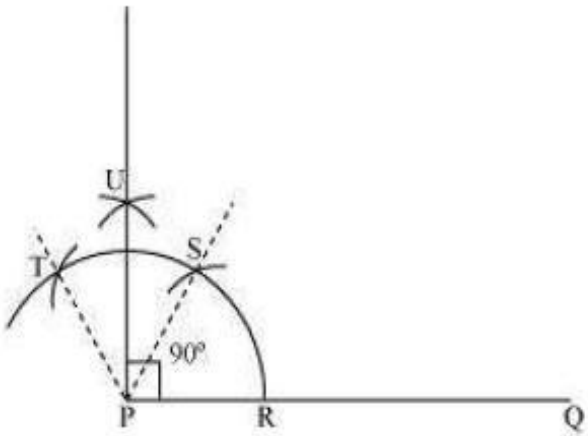
(v) Join PU, which is the required ray making 90° with the given ray PQ.



Justification of Construction:

We can justify the construction, if we can prove ∠UPQ = 90°.

For this, join PS and PT.



We have, ∠SPQ = ∠TPS = 60°. In (iii) and (iv) steps of this construction, PU was drawn as the bisector of ∠TPS.

∴ ∠UPS = $\frac{1}{2}$ ∠TPS $= \frac{1}{2} \times 60^\circ = 30^\circ$

Also, ∠UPQ = ∠SPQ + ∠UPS

= 60° + 30°

= 90°

Question 2:

Construct an angle of 45° at the initial point of a given ray and justify the construction.

Answer:

The below given steps will be followed to construct an angle of 45°.

(i) Take the given ray PQ. Draw an arc of some radius taking point P as its centre, which intersects PQ at R.

(ii) Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at S.

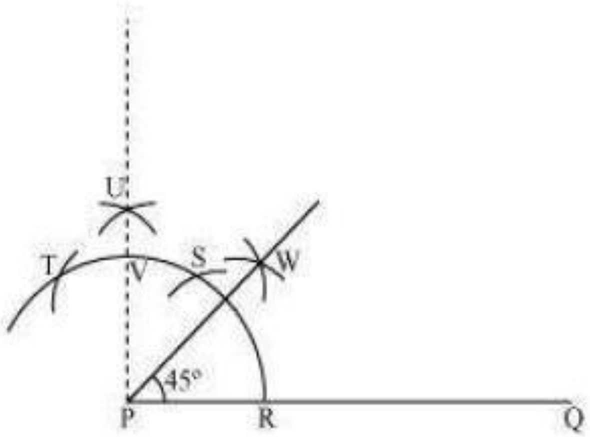
(iii) Taking S as centre and with the same radius as before, draw an arc intersecting the arc at T (see figure).

(iv) Taking S and T as centre, draw an arc of same radius to intersect each other at U.

(v) Join PU. Let it intersect the arc at point V.

(vi) From R and V, draw arcs with radius more than $\frac{1}{2}$ RV to intersect each other at W. Join PW.

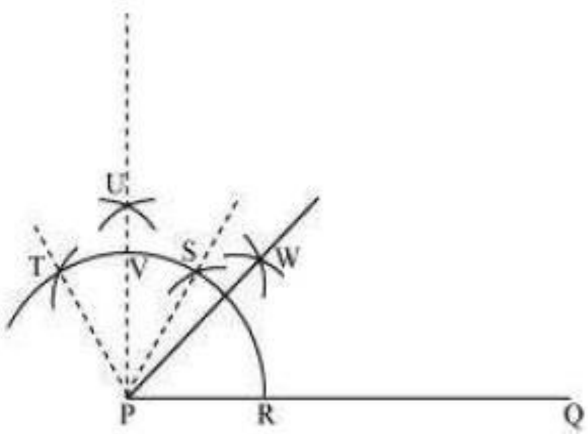
PW is the required ray making 45° with PQ.



Justification of Construction:

We can justify the construction, if we can prove ∠WPQ = 45°.

For this, join PS and PT.



We have, ∠SPQ = ∠TPS = 60°. In (iii) and (iv) steps of this construction, PU was drawn as the bisector of ∠TPS.

∴ ∠UPS = $\frac{1}{2}$ ∠TPS $= \frac{60^\circ}{2} = 30^\circ$

Also, ∠UPQ = ∠SPQ + ∠UPS

= 60° + 30°

= 90°

In step (vi) of this construction, PW was constructed as the bisector of ∠UPQ.

$\therefore \angle WPQ = \frac{1}{2}\angle UPQ = \frac{90^\circ}{2} = 45^\circ$

Question 3:

Construct the angles of the following measurements:

(i) 30° (ii) $22\frac{1}{2}^\circ$ (iii) 15°

Answer:

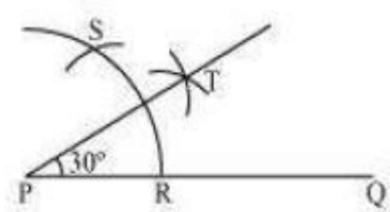
(i)30°

The below given steps will be followed to construct an angle of 30°.

Step I: Draw the given ray PQ. Taking P as centre and with some radius, draw an arc of a circle which intersects PQ at R.

Step II: Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at point S.

Step III: Taking R and S as centre and with radius more than $\frac{1}{2}$ RS, draw arcs to intersect each other at T. Join PT which is the required ray making 30° with the given ray PQ.



(ii) $22\frac{1}{2}^\circ$

The below given steps will be followed to construct an angle of $22\frac{1}{2}^\circ$.

(1) Take the given ray PQ. Draw an arc of some radius, taking point P as its centre, which intersects PQ at R.

(2) Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at S.

(3) Taking S as centre and with the same radius as before, draw an arc intersecting the arc at T (see figure).

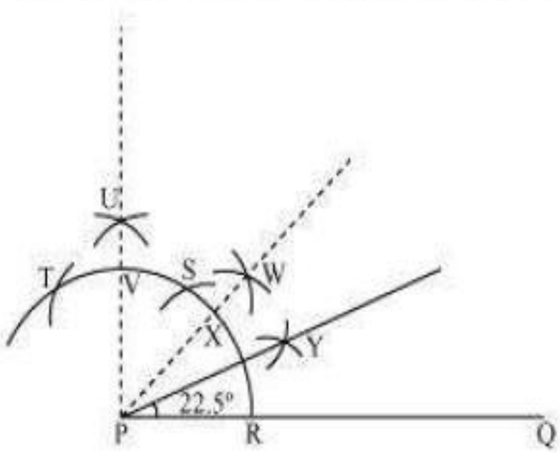
(4) Taking S and T as centre, draw an arc of same radius to intersect each other at U.

(5) Join PU. Let it intersect the arc at point V.

(6) From R and V, draw arcs with radius more than $\frac{1}{2}$ RV to intersect each other at W. Join PW.

(7) Let it intersect the arc at X. Taking X and R as centre and radius more than $\frac{1}{2}$ RX, draw arcs to intersect each other at Y.

Joint PY which is the required ray making $22\frac{1}{2}°$ with the given ray PQ.



(iii) 15°

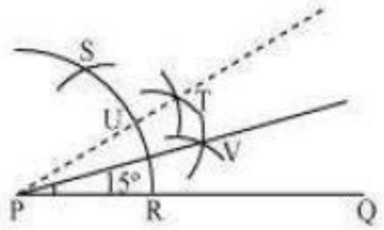
The below given steps will be followed to construct an angle of 15°.

Step I: Draw the given ray PQ. Taking P as centre and with some radius, draw an arc of a circle which intersects PQ at R.

Step II: Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at point S.

Step III: Taking R and S as centre and with radius more than $\frac{1}{2}$ RS, draw arcs to intersect each other at T. Join PT.

Step IV: Let it intersect the arc at U. Taking U and R as centre and with radius more than $\frac{1}{2}$ RU, draw an arc to intersect each other at V. Join PV which is the required ray making 15° with the given ray PQ.



Question 4:

Construct the following angles and verify by measuring them by a protractor:

(i) 75° (ii) 105° (iii) 135°

Answer:

(i) 75°

The below given steps will be followed to construct an angle of 75°.

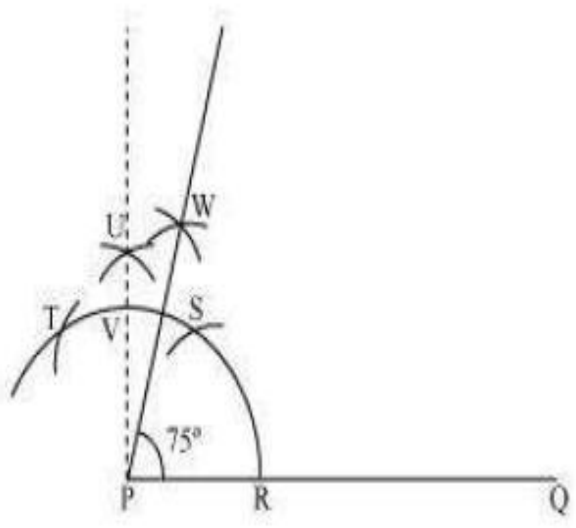
(1) Take the given ray PQ. Draw an arc of some radius taking point P as its centre, which intersects PQ at R.

(2) Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at S.

(3) Taking S as centre and with the same radius as before, draw an arc intersecting the arc at T (see figure).

(4) Taking S and T as centre, draw an arc of same radius to intersect each other at U.

(5) Join PU. Let it intersect the arc at V. Taking S and V as centre, draw arcs with radius more than $\frac{1}{2}$ SV. Let those intersect each other at W. Join PW which is the required ray making 75° with the given ray PQ.



The angle so formed can be measured with the help of a protractor. It comes to be 75°.

(ii) 105°

The below given steps will be followed to construct an angle of 105°.

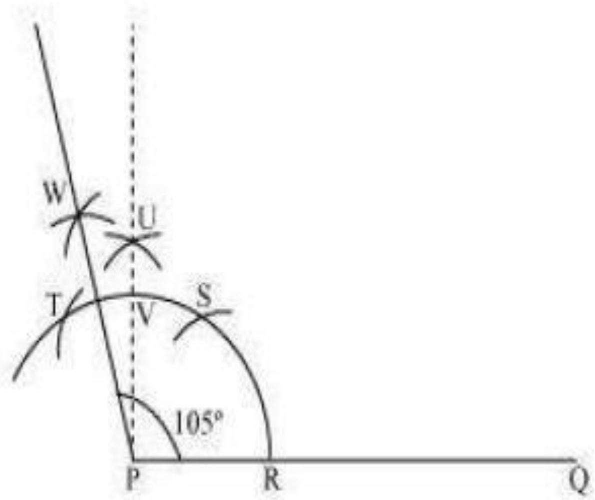
(1) Take the given ray PQ. Draw an arc of some radius taking point P as its centre, which intersects PQ at R.

(2) Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at S.

(3) Taking S as centre and with the same radius as before, draw an arc intersecting the arc at T (see figure).

(4) Taking S and T as centre, draw an arc of same radius to intersect each other at U.

(5) Join PU. Let it intersect the arc at V. Taking T and V as centre, draw arcs with radius more than $\frac{1}{2}$ TV. Let these arcs intersect each other at W. Join PW which is the required ray making 105° with the given ray PQ.



The angle so formed can be measured with the help of a protractor. It comes to be 105°.

(iii) 135°

The below given steps will be followed to construct an angle of 135°.

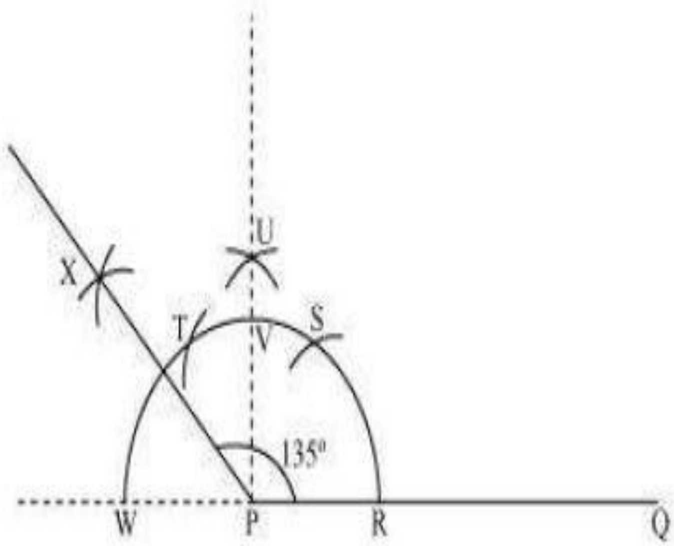
(1) Take the given ray PQ. Extend PQ on the opposite side of Q. Draw a semi-circle of some radius taking point P as its centre, which intersects PQ at R and W.

(2) Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at S.

(3) Taking S as centre and with the same radius as before, draw an arc intersecting the arc at T (see figure).

(4) Taking S and T as centre, draw an arc of same radius to intersect each other at U.

(5) Join PU. Let it intersect the arc at V. Taking V and W as centre and with radius more than $\frac{1}{2}$ VW, draw arcs to intersect each other at X. Join PX, which is the required ray making 135°with the given line PQ.



The angle so formed can be measured with the help of a protractor. It comes to be 135°.

Question 5:

Construct an equilateral triangle, given its side and justify the construction

Answer:

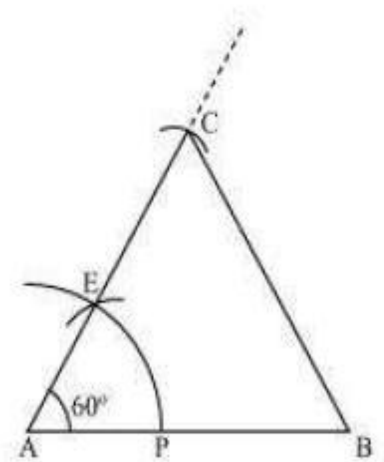
Let us draw an equilateral triangle of side 5 cm. We know that all sides of an equilateral triangle are equal. Therefore, all sides of the equilateral triangle will be 5 cm. We also know that each angle of an equilateral triangle is 60°.

The below given steps will be followed to draw an equilateral triangle of 5 cm side.

Step I: Draw a line segment AB of 5 cm length. Draw an arc of some radius, while taking A as its centre. Let it intersect AB at P.

Step II: Taking P as centre, draw an arc to intersect the previous arc at E. Join AE.

Step III: Taking A as centre, draw an arc of 5 cm radius, which intersects extended line segment AE at C. Join AC and BC. ΔABC is the required equilateral triangle of side 5 cm.



Justification of Construction:

We can justify the construction by showing ABC as an equilateral triangle i.e., AB = BC = AC = 5 cm and ∠A = ∠B = ∠C = 60°.

In ΔABC, we have AC = AB = 5 cm and ∠A = 60°.

Since AC = AB,

∠B = ∠C (Angles opposite to equal sides of a triangle)

In ΔABC,

∠A + ∠B + □C = 180° (Angle sum property of a triangle)

□ 60° + □C + □C = 180°

□ 60° + 2 □C = 180°

□ 2 □C = 180° − 60° = 120°

□ □C = 60°

□ □B = □C = 60°

We have, □A = □B = □C = 60° ... (1)

□ □A = □B and □A = □C

□ BC = AC and BC = AB (Sides opposite to equal angles of a triangle)

□ AB = BC = AC = 5 cm ... (2)

From equations (1) and (2), ΔABC is an equilateral triangle.

Exercise 11.2

Question 1:

Construct a triangle ABC in which BC = 7 cm, ☐B = 75° and AB + AC = 13 cm.

Answer:

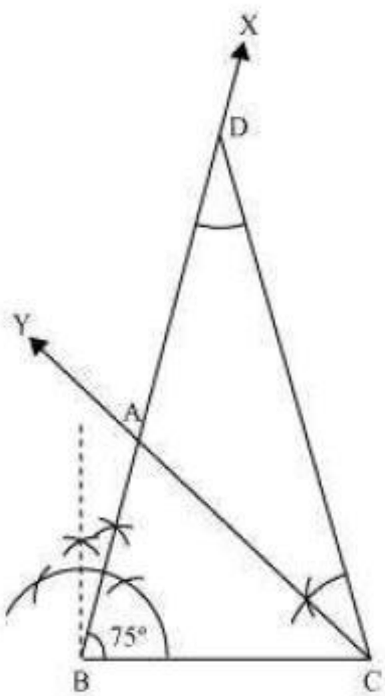
The below given steps will be followed to construct the required triangle.

Step I: Draw a line segment BC of 7 cm. At point B, draw an angle of 75°, say ☐XBC.

Step II: Cut a line segment BD = 13 cm (that is equal to AB + AC) from the ray BX.

Step III: Join DC and make an angle DCY equal to ☐BDC.

Step IV: Let CY intersect BX at A. ΔABC is the required triangle.



Question 2:

Construct a triangle ABC in which BC = 8 cm, ☐B = 45° and AB − AC = 3.5 cm.

Answer:

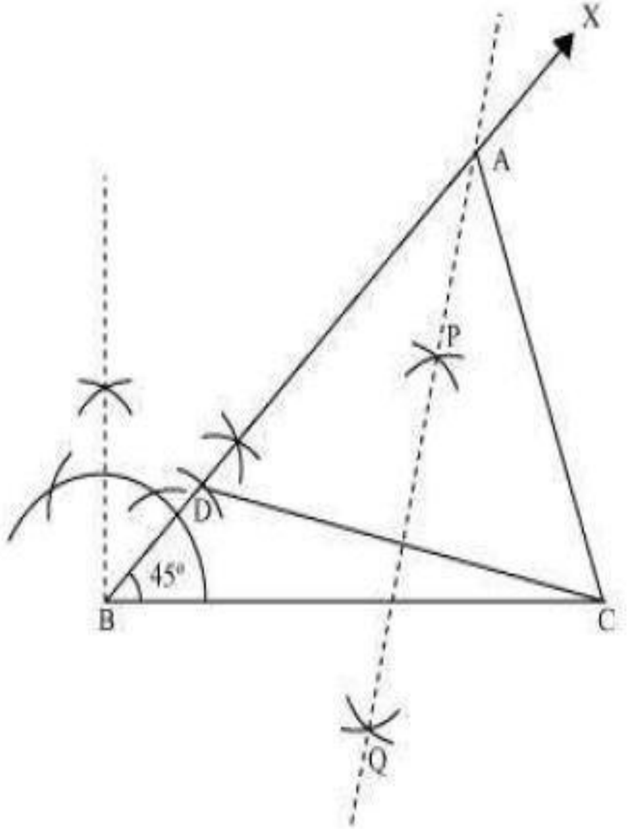
The below given steps will be followed to draw the required triangle.

Step I: Draw the line segment BC = 8 cm and at point B, make an angle of 45°, say ☐XBC.

Step II: Cut the line segment BD = 3.5 cm (equal to AB − AC) on ray BX.

Step III: Join DC and draw the perpendicular bisector PQ of DC.

Step IV: Let it intersect BX at point A. Join AC. ΔABC is the required triangle.



Question 3:

Construct a triangle PQR in which QR = 6 cm, □Q = 60º and PR − PQ = 2 cm

Answer:

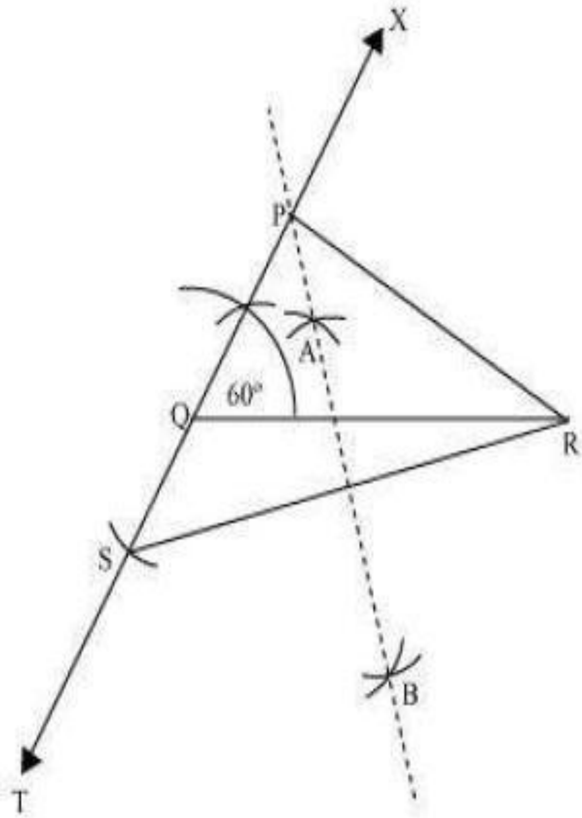
The below given steps will be followed to construct the required triangle.

Step I: Draw line segment QR of 6 cm. At point Q, draw an angle of 60º, say □XQR.

Step II: Cut a line segment QS of 2 cm from the line segment QT extended in the opposite side of line segment XQ. (As PR > PQ and PR − PQ = 2 cm). Join SR.

Step III: Draw perpendicular bisector AB of line segment SR. Let it intersect QX at point P. Join PQ, PR.

ΔPQR is the required triangle.



Question 4:

Construct a triangle XYZ in which □Y = 30°, □Z = 90° and XY + YZ + ZX = 11 cm.

Answer:

The below given steps will be followed to construct the required triangle.

Step I: Draw a line segment AB of 11 cm.

(As XY + YZ + ZX = 11 cm)

Step II: Construct an angle, □PAB, of 30° at point A and an angle, □QBA, of 90° at point B.

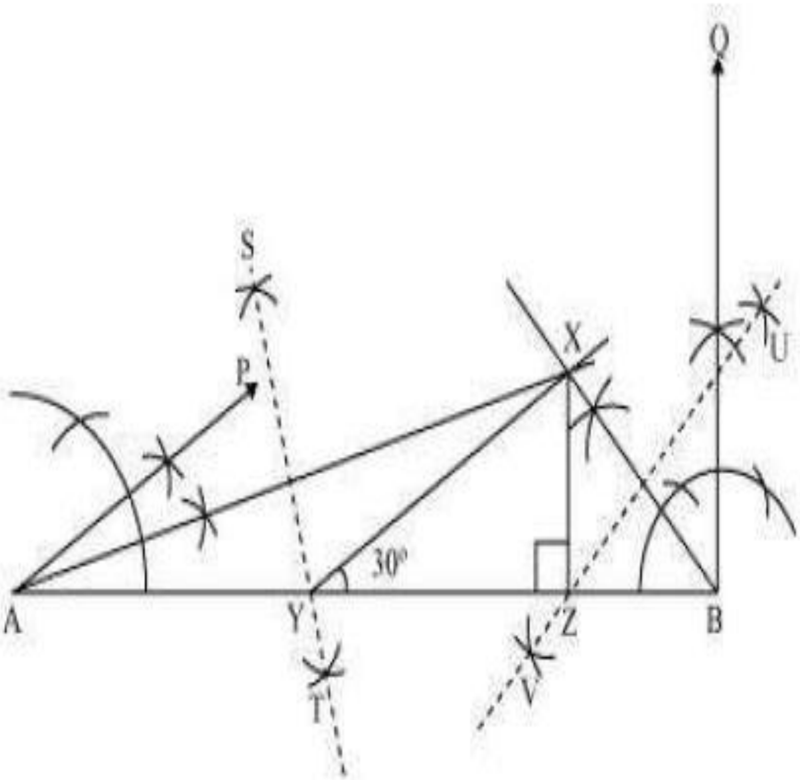
Step III: Bisect □PAB and □QBA. Let these bisectors intersect each other at point X.

Step IV: Draw perpendicular bisector ST of AX and UV of BX.

Step V: Let ST intersect AB at Y and UV intersect AB at Z.

Join XY, XZ.

ΔXYZ is the required triangle.



Question 5:

Construct a right triangle whose base is 12 cm and sum of its hypotenuse and other side is 18 cm.

Answer:

The below given steps will be followed to construct the required triangle.

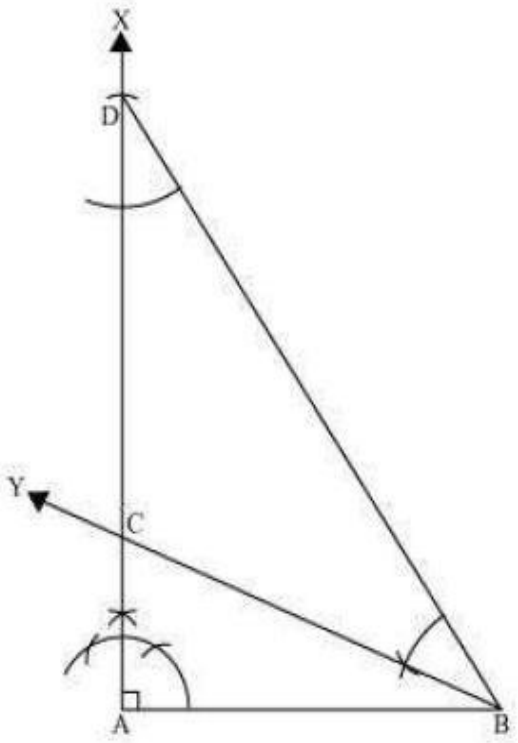
Step I: Draw line segment AB of 12 cm. Draw a ray AX making 90° with AB.

Step II: Cut a line segment AD of 18 cm (as the sum of the other two sides is 18) from ray AX.

Step III: Join DB and make an angle DBY equal to ADB.

Step IV: Let BY intersect AX at C. Join AC, BC.

ΔABC is the required triangle.


X
D
Y
C
A
B

NCERT
Class 9th Maths
Chapter 13: Surface Areas and Volumes

Exercise 13.1

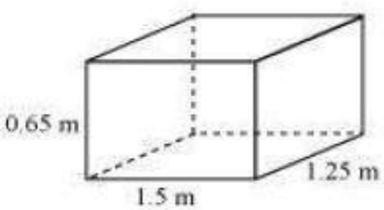
Question 1:

A plastic box 1.5 m long, 1.25 m wide and 65 cm deep, is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine:

(i) The area of the sheet required for making the box.

(ii) The cost of sheet for it, if a sheet measuring 1 m^2 costs Rs 20.

Answer:



It is given that, length (l) of box = 1.5 m

Breadth (b) of box = 1.25 m

Depth (h) of box = 0.65 m

(i) Box is to be open at top.

Area of sheet required

$= 2lh + 2bh + lb$

$= [2 \times 1.5 \times 0.65 + 2 \times 1.25 \times 0.65 + 1.5 \times 1.25]\ m^2$

$= (1.95 + 1.625 + 1.875)\ m^2 = 5.45\ m^2$

(ii) Cost of sheet per m^2 area = Rs 20

Cost of sheet of 5.45 m^2 area = Rs (5.45×20)

= Rs 109

Question 2:

The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs 7.50 per m^2.

Answer:

It is given that

Length (l) of room = 5 m

Breadth (b) of room = 4 m

Height (h) of room = 3 m

It can be observed that four walls and the ceiling of the room are to be white-washed. The floor of the room is not to be white-washed.

Area to be white-washed = Area of walls + Area of ceiling of room

$= 2lh + 2bh + lb$

$= [2 \times 5 \times 3 + 2 \times 4 \times 3 + 5 \times 4]\ m^2$

$= (30 + 24 + 20)\ m^2$

$= 74\ m^2$

Cost of white-washing per m^2 area = Rs 7.50

Cost of white-washing 74 m^2 area = Rs (74 × 7.50)

= Rs 555

Question 3:

The floor of a rectangular hall has a perimeter 250 m. If the cost of panting the four walls at the rate of Rs.10 per m^2 is Rs.15000, find the height of the hall.

[**Hint:** Area of the four walls = Lateral surface area.]

Answer:

Let length, breadth, and height of the rectangular hall be l m, b m, and h m respectively.

Area of four walls = $2lh + 2bh$

$= 2(l + b)\ h$

Perimeter of the floor of hall = $2(l + b)$

= 250 m

$\therefore$ Area of four walls = $2(l + b)\ h = 250h\ m^2$

Cost of painting per m^2 area = Rs 10

Cost of painting $250h\ m^2$ area = Rs ($250h \times 10$) = Rs $2500h$

However, it is given that the cost of paining the walls is Rs 15000.

$\therefore 15000 = 2500h$

$h = 6$

Therefore, the height of the hall is 6 m.

Question 4:

The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Answer:

Total surface area of one brick = $2(lb + bh + lh)$

= [2(22.5 ×10 + 10 × 7.5 + 22.5 × 7.5)] cm^2

= 2(225 + 75 + 168.75) cm^2

= (2 × 468.75) cm^2

= 937.5 cm^2

Let n bricks can be painted out by the paint of the container.

Area of n bricks = (n ×937.5) cm^2 = 937.5n cm^2

Area that can be painted by the paint of the container = 9.375 m^2 = 93750 cm^2

∴ 93750 = 937.5n

n = 100

Therefore, 100 bricks can be painted out by the paint of the container.

Question 5:

A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

(i) Which box has the greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much?

Answer:

(i) Edge of cube = 10 cm

Length (l) of box = 12.5 cm

Breadth (b) of box = 10 cm

Height (h) of box = 8 cm

Lateral surface area of cubical box = 4(edge)^2

= 4(10 cm)^2

= 400 cm^2

Lateral surface area of cuboidal box = $2[lh + bh]$

$= [2(12.5 \times 8 + 10 \times 8)]\ cm^2$

$= (2 \times 180)\ cm^2$

$= 360\ cm^2$

Clearly, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box.

Lateral surface area of cubical box − Lateral surface area of cuboidal box = $400\ cm^2 - 360\ cm^2 = 40\ cm^2$

Therefore, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box by $40\ cm^2$.

(ii) Total surface area of cubical box = $6(edge)^2 = 6(10\ cm)^2 = 600\ cm^2$

Total surface area of cuboidal box

$= 2[lh + bh + lb]$

$= [2(12.5 \times 8 + 10 \times 8 + 12.5 \times 100]\ cm^2$

$= 610\ cm^2$

Clearly, the total surface area of the cubical box is smaller than that of the cuboidal box.

Total surface area of cuboidal box − Total surface area of cubical box = $610\ cm^2 - 600\ cm^2 = 10\ cm^2$

Therefore, the total surface area of the cubical box is smaller than that of the cuboidal box by $10\ cm^2$.

Question 6:

A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

(i) What is the area of the glass?

(ii) How much of tape is needed for all the 12 edges?

Answer:

(i) Length (l) of green house = 30 cm

Breadth (b) of green house = 25 cm

Height (h) of green house = 25 cm

Total surface area of green house

$= 2[lb + lh + bh]$

$= [2(30 \times 25 + 30 \times 25 + 25 \times 25)]\ cm^2$

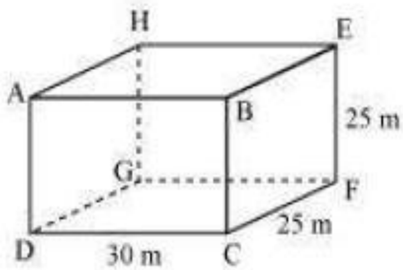
$= [2(750 + 750 + 625)]\ cm^2$

$= (2 \times 2125)\ cm^2$

$= 4250\ cm^2$

Therefore, the area of glass is $4250\ cm^2$.

(ii)



It can be observed that tape is required along side AB, BC, CD, DA, EF, FG, GH, HE, AH, BE, DG, and CF.

Total length of tape $= 4(l + b + h)$

$= [4(30 + 25 + 25)]$ cm

$= 320$ cm

Therefore, 320 cm tape is required for all the 12 edges.

Question 7:

Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs 4 for $1000\ cm^2$, find the cost of cardboard required for supplying 250 boxes of each kind.

Answer:

Length (l_1) of bigger box = 25 cm

Breadth (b_1) of bigger box = 20 cm

Height (h_1) of bigger box = 5 cm

Total surface area of bigger box $= 2(lb + lh + bh)$

$= [2(25 \times 20 + 25 \times 5 + 20 \times 5)]\ cm^2$

$= [2(500 + 125 + 100)]\ cm^2$

$= 1450\ cm^2$

Extra area required for overlapping $= \left(\frac{1450 \times 5}{100}\right) cm^2$

$= 72.5\ cm^2$

While considering all overlaps, total surface area of 1 bigger box

$= (1450 + 72.5)\ cm^2 = 1522.5\ cm^2$

Area of cardboard sheet required for 250 such bigger boxes

$= (1522.5 \times 250)\ cm^2 = 380625\ cm^2$

Similarly, total surface area of smaller box $= [2(15 \times 12 + 15 \times 5 + 12 \times 5]\ cm^2$

$= [2(180 + 75 + 60)]\ cm^2$

$= (2 \times 315)\ cm^2$

$= 630\ cm^2$

Therefore, extra area required for overlapping $= \left(\frac{630 \times 5}{100}\right) cm^2 = 31.5\ cm^2$

Total surface area of 1 smaller box while considering all overlaps

$= (630 + 31.5)\ cm^2 = 661.5\ cm^2$

Area of cardboard sheet required for 250 smaller boxes $= (250 \times 661.5)\ cm^2$

$= 165375\ cm^2$

Total cardboard sheet required $= (380625 + 165375)\ cm^2$

$= 546000\ cm^2$

Cost of $1000\ cm^2$ cardboard sheet = Rs 4

Cost of $546000\ cm^2$ cardboard sheet $= \text{Rs} \left(\frac{546000 \times 4}{1000}\right) = \text{Rs } 2184$

Therefore, the cost of cardboard sheet required for 250 such boxes of each kind will be Rs 2184.

Question 8:

Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

Answer:

Length (l) of shelter = 4 m

Breadth (b) of shelter = 3 m

Height (h) of shelter = 2.5 m

Tarpaulin will be required for the top and four wall sides of the shelter.

Area of Tarpaulin required = $2(lh + bh) + l\,b$

$= [2(4 \times 2.5 + 3 \times 2.5) + 4 \times 3]\ m^2$

$= [2(10 + 7.5) + 12]\ m^2$

$= 47\ m^2$

Therefore, $47\ m^2$ tarpaulin will be required.

Exercise 13.2

Question 1:

The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder. Assume π = $\frac{22}{7}$

Answer:

Height (h) of cylinder = 14 cm

Let the diameter of the cylinder be d.

Curved surface area of cylinder = 88 cm^2

⇒ $2\pi rh$ = 88 cm^2 (r is the radius of the base of the cylinder)

⇒ πdh = 88 cm^2 (d = $2r$)

⇒ $\frac{22}{7} \times d \times 14 \text{ cm} = 88 \text{ cm}^2$

⇒ d = 2 cm

Therefore, the diameter of the base of the cylinder is 2 cm.

Question 2:

It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square meters of the sheet are required for the same? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Height (h) of cylindrical tank = 1 m

Base radius (r) of cylindrical tank $= \left(\frac{140}{2}\right) \text{cm} = 70 \text{ cm} = 0.7 \text{ m}$

Area of sheet required = Total surface area of tank $= 2\pi r(r+h)$

$$= \left[2\times\frac{22}{7}\times 0.7(0.7+1)\right]\ \text{m}^2$$

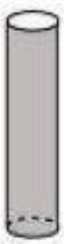
$$= (4.4\times 1.7)\ \text{m}^2$$

$$= 7.48\ \text{m}^2$$

Therefore, it will require 7.48 m^2 area of sheet.

Question 3:

A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm.



(i) Inner curved surface area,

(ii) Outer curved surface area,

(iii) Total surface area. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Inner radius (r_1) of cylindrical pipe $= \left(\frac{4}{2}\right)\text{cm} = 2\ \text{cm}$

Outer radius (r_2) of cylindrical pipe $= \left(\frac{4.4}{2}\right)\text{cm} = 2.2\ \text{cm}$

Height (h) of cylindrical pipe = Length of cylindrical pipe = 77 cm

(i) CSA of inner surface of pipe $= 2\pi r_1 h$

$$= \left(2\times\frac{22}{7}\times 2\times 77\right)\ \text{cm}^2$$

$$= 968\ \text{cm}^2$$

(ii) CSA of outer surface of pipe $= 2\pi r_2 h$

$= \left(2 \times \frac{22}{7} \times 2.2 \times 77\right) \text{ cm}^2$

$= (22 \times 22 \times 2.2) \text{ cm}^2$

$= 1064.8 \text{ cm}^2$

(iii) Total surface area of pipe = CSA of inner surface + CSA of outer surface + Area of both circular ends of pipe

$= 2\pi r_1 h + 2\pi r_2 h + 2\pi\left(r_2^2 - r_1^2\right)$

$= \left[968 + 1064.8 + 2\pi\left\{(2.2)^2 - (2)^2\right\}\right] \text{ cm}^2$

$= \left(2032.8 + 2 \times \frac{22}{7} \times 0.84\right) \text{ cm}^2$

$= (2032.8 + 5.28) \text{ cm}^2$

$= 2038.08 \text{ cm}^2$

Therefore, the total surface area of the cylindrical pipe is 2038.08 cm^2.

Question 4:

The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

It can be observed that a roller is cylindrical.

Height (*h*) of cylindrical roller = Length of roller = 120 cm

Radius (*r*) of the circular end of roller = $\left(\frac{84}{2}\right) \text{ cm} = 42 \text{ cm}$

CSA of roller = $2\pi rh$

$= \left(2 \times \frac{22}{7} \times 42 \times 120\right) \text{ cm}^2$

$= 31680 \text{ cm}^2$

Area of field = 500 × CSA of roller

= (500 × 31680) cm^2

= 15840000 cm^2

= 1584 m^2

Question 5:

A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs.12.50 per m^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Height (*h*) cylindrical pillar = 3.5 m

Radius (*r*) of the circular end of pillar = $\frac{50}{2} = 25 \text{cm}$

= 0.25 m

CSA of pillar = $2\pi rh$

$= \left(2 \times \frac{22}{7} \times 0.25 \times 3.5\right) \text{m}^2$

$= (44 \times 0.125) \text{ m}^2$

$= 5.5 \text{ m}^2$

Cost of painting 1 m^2 area = Rs 12.50

Cost of painting 5.5 m^2 area = Rs (5.5 × 12.50)

= Rs 68.75

Therefore, the cost of painting the CSA of the pillar is Rs 68.75.

Question 6:

Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let the height of the circular cylinder be h.

Radius (r) of the base of cylinder = 0.7 m

CSA of cylinder = 4.4 m^2

$2\pi rh$ = 4.4 m^2

$$\left(2\times\frac{22}{7}\times0.7\times h\right)\ \text{m} = 4.4\ \text{m}^2$$

h = 1 m

Therefore, the height of the cylinder is 1 m.

Question 7:

The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find

(i) Its inner curved surface area,

(ii) The cost of plastering this curved surface at the rate of Rs 40 per m^2.

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

Inner radius (r) of circular well $= \left(\frac{3.5}{2}\right)\ \text{m} = 1.75\ \text{m}$

Depth (h) of circular well = 10 m

Inner curved surface area = $2\pi rh$

$$= \left(2\times\frac{22}{7}\times1.75\times10\right)\ \text{m}^2$$

= (44 × 0.25 × 10) m^2

= 110 m^2

Therefore, the inner curved surface area of the circular well is 110 m^2.

Cost of plastering 1 m^2 area = Rs 40

Cost of plastering 100 m^2 area = Rs (110 × 40)

= Rs 4400

Therefore, the cost of plastering the CSA of this well is Rs 4400.

Question 8:

In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Height (h) of cylindrical pipe = Length of cylindrical pipe = 28 m

Radius (r) of circular end of pipe = $\frac{5}{2}$ = 2.5 cm = 0.025 m

CSA of cylindrical pipe = $2\pi rh$

$= \left(2 \times \frac{22}{7} \times 0.025 \times 28\right) \text{ m}^2$

= 4.4 m^2

The area of the radiating surface of the system is 4.4 m^2.

Question 9:

Find

(i) The lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) How much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Height (h) of cylindrical tank = 4.5 m

Radius (r) of the circular end of cylindrical tank = $\left(\frac{4.2}{2}\right) \text{ m} = 2.1 \text{ m}$

(i) Lateral or curved surface area of tank = $2\pi rh$

$= \left(2 \times \frac{22}{7} \times 2.1 \times 4.5\right) \text{ m}^2$

$= (44 \times 0.3 \times 4.5)\ m^2$

$= 59.4\ m^2$

Therefore, CSA of tank is $59.4\ m^2$.

(ii) Total surface area of tank = $2\pi r\,(r + h)$

$$= \left[2 \times \frac{22}{7} \times 2.1 \times (2.1 + 4.5)\right] m^2$$

$= (44 \times 0.3 \times 6.6)\ m^2$

$= 87.12\ m^2$

Let A m^2 steel sheet be actually used in making the tank.

$$\therefore A\left(1 - \frac{1}{12}\right) = 87.12\ m^2$$

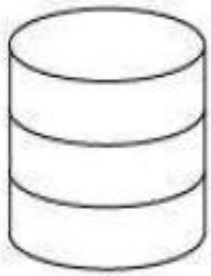
$$\Rightarrow A = \left(\frac{12}{11} \times 87.12\right) m^2$$

$$\Rightarrow A = 95.04\ m^2$$

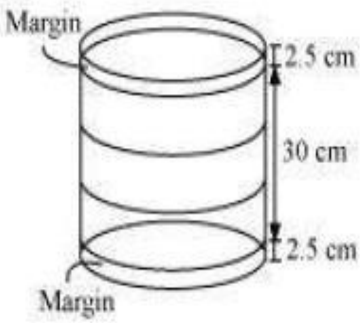
Therefore, $95.04\ m^2$ steel was used in actual while making such a tank.

Question 10:

In the given figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade. $\left[\text{Assume } \pi = \frac{22}{7}\right]$



Answer:



Height (h) of the frame of lampshade = (2.5 + 30 + 2.5) cm = 35 cm

Radius (r) of the circular end of the frame of lampshade = $\left(\frac{20}{2}\right)$ cm = 10 cm

Cloth required for covering the lampshade = $2\pi rh$

$= \left(2 \times \frac{22}{7} \times 10 \times 35\right) \text{cm}^2$

= 2200 cm^2

Hence, for covering the lampshade, 2200 cm^2 cloth will be required.

Question 11:

The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of the circular end of cylindrical penholder = 3 cm

Height (h) of penholder = 10.5 cm

Surface area of 1 penholder = CSA of penholder + Area of base of penholder

$= 2\pi rh + \pi r^2$

$$= \left[2 \times \frac{22}{7} \times 3 \times 10.5 + \frac{22}{7} \times (3)^2\right] \text{cm}^2$$

$$= \left(132 \times 1.5 + \frac{198}{7}\right) \text{cm}^2$$

$$= \left(198 + \frac{198}{7}\right) \text{cm}^2$$

$$= \frac{1584}{7} \text{cm}^2$$

Area of cardboard sheet used by 1 competitor $= \frac{1584}{7} \text{cm}^2$

Area of cardboard sheet used by 35 competitors

$= \left(\frac{1584}{7} \times 35\right) \text{cm}^2 = 7920 \text{ cm}^2$

Therefore, 7920 cm^2 cardboard sheet will be bought.

Exercise 13.3

Question 1:

Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of the base of cone = $\left(\frac{10.5}{2}\right)$ cm = 5.25 cm

Slant height (l) of cone = 10 cm

CSA of cone = πrl

$$= \left(\frac{22}{7} \times 5.25 \times 10\right) \text{ cm}^2 = (22 \times 0.75 \times 10) \text{ cm}^2 = 165 \text{ cm}^2$$

Therefore, the curved surface area of the cone is 165 cm^2.

Question 2:

Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of the base of cone = $\left(\frac{24}{2}\right)$ m = 12 m

Slant height (l) of cone = 21 m

Total surface area of cone = πr(r + l)

$$= \left[\frac{22}{7} \times 12 \times (12 + 21)\right] \text{ m}^2$$

$$= \left(\frac{22}{7} \times 12 \times 33\right) \text{ m}^2$$

$$= 1244.57 \text{ m}^2$$

Question 3:

Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find

(i) radius of the base and (ii) total surface area of the cone.

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Slant height (l) of cone = 14 cm

Let the radius of the circular end of the cone be r.

We know, CSA of cone = $\pi r l$

$$(308)\ \text{cm}^2 = \left(\frac{22}{7} \times r \times 14\right)\ \text{cm}$$

$$\Rightarrow r = \left(\frac{308}{44}\right)\ \text{cm} = 7\ \text{cm}$$

Therefore, the radius of the circular end of the cone is 7 cm.

(ii) Total surface area of cone = CSA of cone + Area of base

$= \pi r l + \pi r^2$

$$= \left[308 + \frac{22}{7} \times (7)^2\right]\ \text{cm}^2$$

$$= (308 + 154)\ \text{cm}^2$$

$$= 462\ \text{cm}^2$$

Therefore, the total surface area of the cone is 462 cm^2.

Question 4:

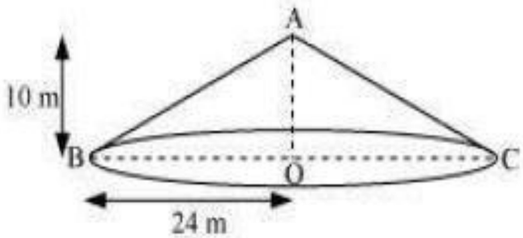
A conical tent is 10 m high and the radius of its base is 24 m. Find

(i) slant height of the tent

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs 70.

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:



(i) Let ABC be a conical tent.

Height (h) of conical tent = 10 m

Radius (r) of conical tent = 24 m

Let the slant height of the tent be l.

In ΔABO,

$AB^2 = AO^2 + BO^2$

$l^2 = h^2 + r^2$

$= (10 \text{ m})^2 + (24 \text{ m})^2$

$= 676 \text{ m}^2$

$\therefore l = 26$ m

Therefore, the slant height of the tent is 26 m.

(ii) CSA of tent = πrl

$= \left(\frac{22}{7} \times 24 \times 26\right) \text{ m}^2$

$= \frac{13728}{7} \text{ m}^2$

Cost of 1 m^2 canvas = Rs 70

Cost of $\frac{13728}{7} \text{ m}^2$ canvas = Rs $\left(\frac{13728}{7} \times 70\right)$

= Rs 137280

Therefore, the cost of the canvas required to make such a tent is Rs 137280.

Question 5:

What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use π = 3.14]

Answer:

Height (h) of conical tent = 8 m

Radius (r) of base of tent = 6 m

Slant height (l) of tent = $\sqrt{r^2+h^2}$

$=\left(\sqrt{6^2+8^2}\right)\text{ m}=\left(\sqrt{100}\right)\text{ m}=10\text{ m}$

CSA of conical tent = πrl

= $(3.14 \times 6 \times 10)\text{ m}^2$

= 188.4 m^2

Let the length of tarpaulin sheet required be l.

As 20 cm will be wasted, therefore, the effective length will be (l – 0.2 m).

Breadth of tarpaulin = 3 m

Area of sheet = CSA of tent

$[(l - 0.2\text{ m}) \times 3]\text{ m} = 188.4\text{ m}^2$

$l - 0.2\text{ m} = 62.8\text{ m}$

$l = 63\text{ m}$

Therefore, the length of the required tarpaulin sheet will be 63 m.

Question 6:

The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs 210 per 100 m^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Slant height (l) of conical tomb = 25 m

Base radius (r) of tomb = $\frac{14}{2} =$ 7 m

CSA of conical tomb = πrl

$$= \left(\frac{22}{7} \times 7 \times 25\right) \text{ m}^2$$

= 550 m^2

Cost of white-washing 100 m^2 area = Rs 210

Cost of white-washing 550 m^2 area = $\text{Rs}\left(\frac{210 \times 550}{100}\right)$

= Rs 1155

Therefore, it will cost Rs 1155 while white-washing such a conical tomb.

Question 7:

A joker's cap is in the form of right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of conical cap = 7 cm

Height (h) of conical cap = 24 cm

Slant height (l) of conical cap = $\sqrt{r^2 + h^2}$

$$= \left[\sqrt{(7)^2 + (24)^2}\right] \text{ cm} = \left(\sqrt{625}\right) \text{ cm} = 25 \text{ cm}$$

CSA of 1 conical cap = πrl

$$= \left(\frac{22}{7} \times 7 \times 25\right) \text{ cm}^2 = 550 \text{ cm}^2$$

CSA of 10 such conical caps = (10×550) cm^2 = 5500 cm^2

Therefore, 5500 cm^2 sheet will be required.

Question 8:

A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs 12 per m^2, what will be the cost of painting all these cones? (Use π = 3.14 and take $\sqrt{1.04}$ = 1.02).

Answer:

Radius (r) of cone = $\frac{40}{2} = 20 \text{ cm}$ = 0.2 m

Height (h) of cone = 1 m

Slant height (l) of cone = $\sqrt{h^2 + r^2}$

$= \left[\sqrt{(1)^2 + (0.2)^2}\right] \text{ m} = \left(\sqrt{1.04}\right) \text{ m} = 1.02 \text{ m}$

CSA of each cone = πrl

= (3.14 × 0.2 × 1.02) m^2 = 0.64056 m^2

CSA of 50 such cones = (50 × 0.64056) m^2

= 32.028 m^2

Cost of painting 1 m^2 area = Rs 12

Cost of painting 32.028 m^2 area = Rs (32.028 × 12)

= Rs 384.336

= Rs 384.34 (approximately)

Therefore, it will cost Rs 384.34 in painting 50 such hollow cones.

Exercise 13.4

Question 1:

Find the surface area of a sphere of radius:

(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Radius (r) of sphere = 10.5 cm

Surface area of sphere = $4\pi r^2$

$$= \left[4 \times \frac{22}{7} \times (10.5)^2\right] \text{ cm}^2$$

$$= \left(4 \times \frac{22}{7} \times 10.5 \times 10.5\right) \text{ cm}^2$$

$$= (88 \times 1.5 \times 10.5) \text{ cm}^2$$

$$= 1386 \text{ cm}^2$$

Therefore, the surface area of a sphere having radius 10.5cm is 1386 cm^2.

(ii) Radius(r) of sphere = 5.6 cm

Surface area of sphere = $4\pi r^2$

$$= \left[4 \times \frac{22}{7} \times (5.6)^2\right] \text{ cm}^2$$

$$= (88 \times 0.8 \times 5.6) \text{ cm}^2$$

$$= 394.24 \text{ cm}^2$$

Therefore, the surface area of a sphere having radius 5.6 cm is 394.24 cm^2.

(iii) Radius (r) of sphere = 14 cm

Surface area of sphere = $4\pi r^2$

$$= \left[4 \times \frac{22}{7} \times (14)^2\right] \text{ cm}^2$$

$$= (4 \times 44 \times 14) \text{ cm}^2$$

$$= 2464 \text{ cm}^2$$

Therefore, the surface area of a sphere having radius 14 cm is 2464 cm^2.

Question 2:

Find the surface area of a sphere of diameter:

(i) 14 cm (ii) 21 cm (iii) 3.5 m

$\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

(i) Radius (r) of sphere = $\frac{\text{Diameter}}{2} = \left(\frac{14}{2}\right) \text{ cm} = 7 \text{ cm}$

Surface area of sphere = $4\pi r^2$

$= \left(4 \times \frac{22}{7} \times (7)^2\right) \text{ cm}^2$

$= (88 \times 7\ ) \text{cm}^2$

$= 616 \text{ cm}^2$

Therefore, the surface area of a sphere having diameter 14 cm is 616 cm^2.

(ii) Radius (r) of sphere = $\frac{21}{2} = 10.5 \text{ cm}$

Surface area of sphere = $4\pi r^2$

$= \left[4 \times \frac{22}{7} \times (10.5)^2\right] \text{ cm}^2$

$= 1386 \text{ cm}^2$

Therefore, the surface area of a sphere having diameter 21 cm is 1386 cm^2.

(iii) Radius (r) of sphere = $\frac{3.5}{2} = 1.75$ m

Surface area of sphere = $4\pi r^2$

$= \left[4 \times \frac{22}{7} \times (1.75)^2\right] \text{ m}^2$

$= 38.5 \text{ m}^2$

Therefore, the surface area of the sphere having diameter 3.5 m is 38.5 m^2.

Question 3:

Find the total surface area of a hemisphere of radius 10 cm. [Use π = 3.14]

Answer:

Radius (r) of hemisphere = 10 cm

Total surface area of hemisphere = CSA of hemisphere + Area of circular end of hemisphere

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$= \left[3 \times 3.14 \times (10)^2\right] \text{ cm}^2$

$= 942 \text{ cm}^2$

Therefore, the total surface area of such a hemisphere is 942 cm^2.

Question 4:

The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Answer:

Radius (r_1) of spherical balloon = 7 cm

Radius (r_2) of spherical balloon, when air is pumped into it = 14 cm

$$\text{Required ratio} = \frac{\text{Initial surface area}}{\text{Surface area after pumping air into balloon}}$$

$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{7}{14}\right)^2 = \frac{1}{4}$$

Therefore, the ratio between the surface areas in these two cases is 1:4.

Question 5:

A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Inner radius (r) of hemispherical bowl $= \left(\frac{10.5}{2}\right)\text{ cm} = 5.25\text{ cm}$

Surface area of hemispherical bowl = $2\pi r^2$

$= \left[2 \times \frac{22}{7} \times (5.25)^2\right]\text{ cm}^2$

$= 173.25\text{ cm}^2$

Cost of tin-plating 100 cm^2 area = Rs 16

Cost of tin-plating 173.25 cm^2 area $= \text{Rs}\left(\frac{16 \times 173.25}{100}\right)$ = Rs 27.72

Therefore, the cost of tin-plating the inner side of the hemispherical bowl is Rs 27.72.

Question 6:

Find the radius of a sphere whose surface area is 154 cm^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let the radius of the sphere be r.

Surface area of sphere = 154

$\therefore 4\pi r^2 = 154\text{ cm}^2$

$r^2 = \left(\frac{154 \times 7}{4 \times 22}\right)\text{ cm}^2 = \left(\frac{7 \times 7}{2 \times 2}\right)\text{ cm}^2$

$r = \left(\frac{7}{2}\right)\text{ cm} = 3.5\text{ cm}$

Therefore, the radius of the sphere whose surface area is 154 cm^2 is 3.5 cm.

Question 7:

The diameter of the moon is approximately one-fourth of the diameter of the earth. Find the ratio of their surface area.

Answer:

Let the diameter of earth be *d*. Therefore, the diameter of moon will be $\frac{d}{4}$.

Radius of earth = $\frac{d}{2}$

Radius of moon = $\frac{1}{2} \times \frac{d}{4} = \frac{d}{8}$

Surface area of moon = $4\pi\left(\frac{d}{8}\right)^2$

Surface area of earth = $4\pi\left(\frac{d}{2}\right)^2$

Required ratio $= \frac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2}$

$= \frac{4}{64} = \frac{1}{16}$

Therefore, the ratio between their surface areas will be 1:16.

Question 8:

A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Inner radius of hemispherical bowl = 5 cm

Thickness of the bowl = 0.25 cm

∴ Outer radius (r) of hemispherical bowl = (5 + 0.25) cm

= 5.25 cm

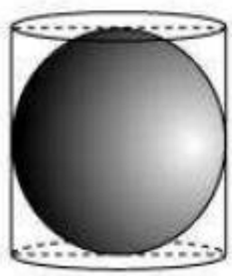
Outer CSA of hemispherical bowl = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times (5.25 \text{ cm})^2 = 173.25 \text{ cm}^2$$

Therefore, the outer curved surface area of the bowl is 173.25 cm^2.

Question 9:

A right circular cylinder just encloses a sphere of radius r (see figure). Find

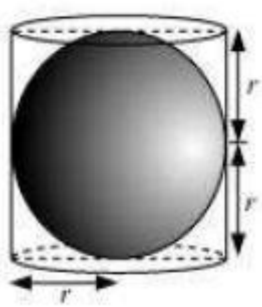


(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii).

Answer:



(i) Surface area of sphere = $4\pi r^2$

(ii) Height of cylinder = $r + r = 2r$

Radius of cylinder = r

CSA of cylinder = $2\pi rh$

= $2\pi r(2r)$

= $4\pi r^2$

(iii) $\text{Required ratio} = \dfrac{\text{Surface area of sphere}}{\text{CSA of cylinder}}$

$= \dfrac{4\pi r^2}{4\pi r^2}$

$= \dfrac{1}{1}$

Therefore, the ratio between these two surface areas is 1:1.

Exercise 13.5

Question 1:

A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?

Answer:

Matchbox is a cuboid having its length (l), breadth (b), height (h) as 4 cm, 2.5 cm, and 1.5 cm.

Volume of 1 match box = $l \times b \times h$

= $(4 \times 2.5 \times 1.5)$ cm^3 = 15 cm^3

Volume of 12 such matchboxes = (15×12) cm^3

= 180 cm^3

Therefore, the volume of 12 match boxes is 180 cm^3.

Question 2:

A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? (1 m^3 = 1000l)

Answer:

The given cuboidal water tank has its length (l) as 6 m, breadth (b) as 5 m, and height (h) as 4.5 m.

Volume of tank = $l \times b \times h$

= $(6 \times 5 \times 4.5)$ m^3 = 135 m^3

Amount of water that 1 m^3 volume can hold = 1000 litres

Amount of water that 135 m^3 volume can hold = (135×1000) litres

= 135000 litres

Therefore, such tank can hold up to 135000 litres of water.

Question 3:

A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Answer:

Let the height of the cuboidal vessel be h.

Length (l) of vessel = 10 m

Width (b) of vessel = 8 m

Volume of vessel = 380 m^3

$\therefore l \times b \times h = 380$

[(10) (8) h] m^2= 380 m^3

$h = 4.75$ m

Therefore, the height of the vessel should be 4.75 m.

Question 4:

Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs 30 per m^3.

Answer:

The given cuboidal pit has its length (l) as 8 m, width (b) as 6 m, and depth (h)as 3 m.

Volume of pit = $l \times b \times h$

= (8 × 6 × 3) m^3 = 144 m^3

Cost of digging per m^3 volume = Rs 30

Cost of digging 144 m^3 volume = Rs (144 × 30) = Rs 4320

Question 5:

The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Answer:

Let the breadth of the tank be b m.

Length (l) and depth (h) of tank is 2.5 m and 10 m respectively.

Volume of tank = $l \times b \times h$

= (2.5 × b × 10) m^3

= 25b m^3

Capacity of tank = 25b m^3 = 25000 b litres

$\therefore$ 25000 b = 50000

$\Rightarrow b = 2$

Therefore, the breadth of the tank is 2 m.

Question 6:

A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Answer:

The given tank is cuboidal in shape having its length (l) as 20 m, breadth (b) as 15 m, and height (h) as 6 m.

Capacity of tank = $l \times b \times h$

= $(20 \times 15 \times 6)\ m^3 = 1800\ m^3$ = 1800000 litres

Water consumed by the people of the village in 1 day = (4000 × 150) litres

= 600000 litres

Let water in this tank last for n days.

Water consumed by all people of village in n days = Capacity of tank

$n \times 600000 = 1800000$

$n = 3$

Therefore, the water of this tank will last for 3 days.

Question 7:

A godown measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

Answer:

The godown has its length (l_1) as 40 m, breadth (b_1) as 25 m, height (h_1) as 10 m, while the wooden crate has its length (l_2) as 1.5 m, breadth (b_2) as 1.25 m, and height (h_2) as 0.5 m.

Therefore, volume of godown = $l_1 \times b_1 \times h_1$

= $(40 \times 25 \times 10)\ m^3$

= $10000\ m^3$

Volume of 1 wooden crate = $l_2 \times b_2 \times h_2$

= $(1.5 \times 1.25 \times 0.5)\ m^3$

= $0.9375\ m^3$

Let n wooden crates can be stored in the godown.

Therefore, volume of n wooden crates = Volume of godown

$0.9375 \times n = 10000$

$$n = \frac{10000}{0.9375} = 10666.66$$

Therefore, 10666 wooden crates can be stored in the godown.

Question 8:

A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Answer:

Side (a) of cube = 12 cm

Volume of cube = $(a)^3 = (12 \text{ cm})^3 = 1728 \text{ cm}^3$

Let the side of the smaller cube be a_1.

Volume of 1 smaller cube $= \left(\frac{1728}{8}\right) \text{cm}^3 = 216 \text{ cm}^3$

$$(a_1)^3 = 216 \text{ cm}^3$$

$\Rightarrow a_1 = 6$ cm

Therefore, the side of the smaller cubes will be 6 cm.

Ratio between surface areas of cubes $= \frac{\text{Surface area of bigger cube}}{\text{Surface area of smaller cube}}$

$$= \frac{6a^2}{6a_1^2} = \frac{(12)^2}{(6)^2}$$

$$= \frac{4}{1}$$

Therefore, the ratio between the surface areas of these cubes is 4:1.

Question 9:

A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Answer:

Rate of water flow = 2 km per hour

$= \left(\frac{2000}{60}\right)$ m/min

$= \left(\frac{100}{3}\right)$ m/min

Depth (h) of river = 3 m

Width (b) of river = 40 m

Volume of water flowed in 1 min $= \left(\frac{100}{3} \times 40 \times 3\right)$ m^3 = 4000 m^3

Therefore, in 1 minute, 4000 m^3 water will fall in the sea.

Exercise 13.6

Question 1:

The circumference of the base of cylindrical vessel is 132 cm and its height is 25 cm.

How many litres of water can it hold? (1000 cm^3 = 1*l*) $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let the radius of the cylindrical vessel be *r*.

Height (*h*) of vessel = 25 cm

Circumference of vessel = 132 cm

$2\pi r = 132$ cm

$r = \left(\frac{132 \times 7}{2 \times 22}\right) \text{ cm} = 21 \text{ cm}$

Volume of cylindrical vessel = $\pi r^2 h$

$= \left[\frac{22}{7} \times (21)^2 \times 25\right] \text{ cm}^3$

= 34650 cm^3

$= \left(\frac{34650}{1000}\right) \text{ litres} \qquad \left[\because 1 \text{ litre} = 1000 \text{ cm}^3\right]$

= 34.65 litres

Therefore, such vessel can hold 34.65 litres of water.

Question 2:

The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Inner radius (r_1) of cylindrical pipe = $\left(\frac{24}{2}\right) \text{ cm} = 12 \text{ cm}$

Outer radius (r_2) of cylindrical pipe = $\left(\frac{28}{2}\right)$ cm = 14 cm

Height (h) of pipe = Length of pipe = 35 cm

Volume of pipe = $\pi\left(r_2^2 - r_1^2\right)h$

$= \left[\frac{22}{7} \times \left(14^2 - 12^2\right) \times 35\right] \text{cm}^3$

$= 110 \times 52 \text{ cm}^3$

$= 5720 \text{ cm}^3$

Mass of 1 cm^3 wood = 0.6 g

Mass of 5720 cm^3 wood = (5720 × 0.6) g

= 3432 g

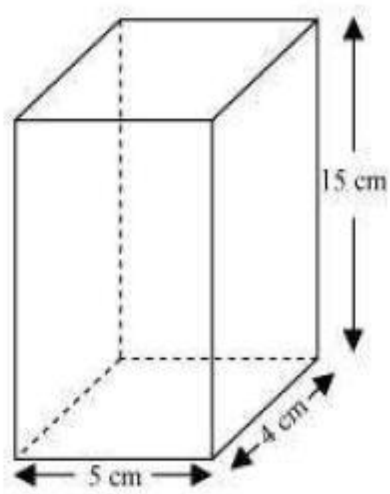
= 3.432 kg

Question 3:

A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

The tin can will be cuboidal in shape while the plastic cylinder will be cylindrical in shape.



Length (l) of tin can = 5 cm

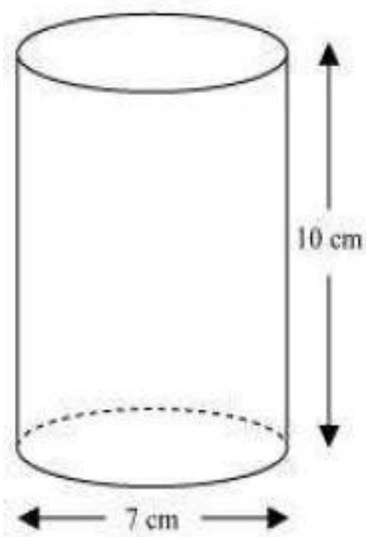
Breadth (b) of tin can = 4 cm

Height (h) of tin can = 15 cm

Capacity of tin can = $l \times b \times h$

= $(5 \times 4 \times 15)$ cm^3

= 300 cm^3



Radius (r) of circular end of plastic cylinder = $\left(\frac{7}{2}\right)$ cm = 3.5 cm

Height (H) of plastic cylinder = 10 cm

Capacity of plastic cylinder = $\pi r^2 H$

$= \left[\frac{22}{7} \times (3.5)^2 \times 10\right] \text{ cm}^3$

$= (11 \times 35) \text{ cm}^3$

$= 385 \text{ cm}^3$

Therefore, plastic cylinder has the greater capacity.

Difference in capacity = (385 − 300) cm^3 = 85 cm^3

Question 4:

If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find (i) radius of its base (ii) its volume. [Use π = 3.14]

Answer:

(i) Height (h) of cylinder = 5 cm

Let radius of cylinder be r.

CSA of cylinder = 94.2 cm^2

$2\pi rh = 94.2 \text{ cm}^2$

$(2 \times 3.14 \times r \times 5) \text{ cm} = 94.2 \text{ cm}^2$

$r = 3 \text{ cm}$

(ii) Volume of cylinder = $\pi r^2 h$

$= (3.14 \times (3)^2 \times 5) \text{ cm}^3$

$= 141.3 \text{ cm}^3$

Question 5:

It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of Rs 20 per m^2, find

(i) Inner curved surface area of the vessel

(ii) Radius of the base

(iii) Capacity of the vessel

$\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

(i) Rs 20 is the cost of painting 1 m^2 area.

Rs 2200 is the cost of painting = $\left(\frac{1}{20}\times 2200\right)\ m^2$ area

= 110 m^2 area

Therefore, the inner surface area of the vessel is 110 m^2.

(ii) Let the radius of the base of the vessel be r.

Height (h) of vessel = 10 m

Surface area = $2\pi rh$ = 110 m^2

$$\Rightarrow \left(2\times\frac{22}{7}\times r\times 10\right)\ m = 110\ m^2$$

$$\Rightarrow r = \left(\frac{7}{4}\right)\ m = 1.75\ m$$

(iii) Volume of vessel = $\pi r^2 h$

$$= \left[\frac{22}{7}\times(1.75)^2\times 10\right]\ m^3$$

= 96.25 m^3

Therefore, the capacity of the vessel is 96.25 m^3 or 96250 litres.

Question 6:

The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let the radius of the circular end be r.

Height (h) of cylindrical vessel = 1 m

Volume of cylindrical vessel = 15.4 litres = 0.0154 m^3

$$\pi r^2 h = 0.0154\ m^3$$

$$\left(\frac{22}{7}\times r^2\times 1\right)\ m = 0.0154\ m^3$$

$\Rightarrow r = 0.07$ m

Total surface area of vessel $= 2\pi r(r+h)$

$$= \left[2 \times \frac{22}{7} \times 0.07(0.07+1)\right] \text{ m}^2$$

$$= 0.44 \times 1.07 \text{ m}^2$$

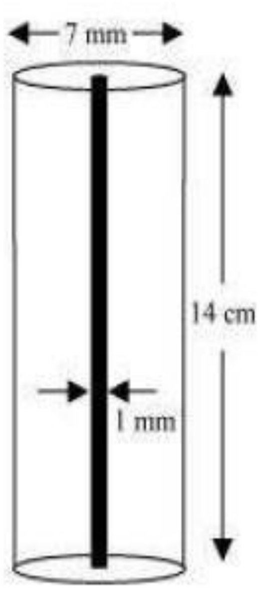
$$= 0.4708 \text{ m}^2$$

Therefore, 0.4708 m^2 of the metal sheet would be required to make the cylindrical vessel.

Question 7:

A lead pencil consists of a cylinder of wood with solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:



Radius (r_1) of pencil $= \left(\frac{7}{2}\right) \text{ mm} = \left(\frac{0.7}{2}\right) \text{ cm} = 0.35$ cm

Radius (r_2) of graphite = $\left(\frac{1}{2}\right)$ mm $= \left(\frac{0.1}{2}\right)$ cm = 0.05 cm

Height (h) of pencil = 14 cm

Volume of wood in pencil = $\pi\left(r_1^2 - r_2^2\right)h$

$$= \left[\frac{22}{7}\left\{(0.35)^2 - (0.05)^2 \times 14\right\}\right] \text{cm}^3$$

$$= \left[\frac{22}{7}(0.1225 - 0.0025) \times 14\right] \text{cm}^3$$

$$= (44 \times 0.12) \text{ cm}^3$$

$$= 5.28 \text{ cm}^3$$

Volume of graphite $= \pi r_2^2 h = \left[\frac{22}{7} \times (0.05)^2 \times 14\right] cm^3$

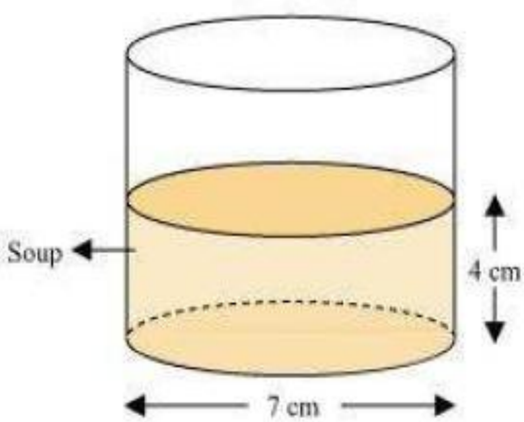
$$= (44 \times 0.0025)\ cm^3$$

$$= 0.11\ cm^3$$

Question 8:

A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:



Radius (r) of cylindrical bowl = $\left(\frac{7}{2}\right)$ cm = 3.5 cm

Height (h) of bowl, up to which bowl is filled with soup = 4 cm

Volume of soup in 1 bowl = $\pi r^2 h$

$=\left(\frac{22}{7}\times(3.5)^2\times 4\right)$ cm^3

= (11 × 3.5 × 4) cm^3

= 154 cm^3

Volume of soup given to 250 patients = (250 × 154) cm^3

= 38500 cm^3

= 38.5 litres.

Exercise 13.7

Question 1:

Find the volume of the right circular cone with

(i) radius 6 cm, height 7 cm

(ii) radius 3.5 cm, height 12 cm

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Radius (r) of cone = 6 cm

Height (h) of cone = 7 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7\right] \text{ cm}^3$$

$$= (12 \times 22) \text{ cm}^3$$

$$= 264 \text{ cm}^3$$

Therefore, the volume of the cone is 264 cm^3.

(ii) Radius (r) of cone = 3.5 cm

Height (h) of cone = 12 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12\right] \text{ cm}^3$$

$$= \left(\frac{1}{3} \times 22 \times \frac{1}{2} \times 3.5 \times 12\right) \text{ cm}^3$$

$$= 154 \text{ cm}^3$$

Therefore, the volume of the cone is 154 cm^3.

Question 2:

Find the capacity in litres of a conical vessel with

(i) radius 7 cm, slant height 25 cm

(ii) height 12 cm, slant height 12 cm

$\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

(i) Radius (r) of cone = 7 cm

Slant height (l) of cone = 25 cm

Height (h) of cone $= \sqrt{l^2 - r^2}$

$= \left(\sqrt{25^2 - 7^2}\right)$ cm

$= 24$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24\right)$ cm^3

$= (154 \times 8)$ cm^3

$= 1232$ cm^3

Therefore, capacity of the conical vessel

$= \left(\frac{1232}{1000}\right)$ litres $\left(1 \text{ litre} = 1000 \text{ cm}^3\right)$

= 1.232 litres

(ii) Height (h) of cone = 12 cm

Slant height (l) of cone = 13 cm

Radius (r) of cone $= \sqrt{l^2 - h^2}$

$= \left(\sqrt{13^2 - 12^2}\right)$ cm

$= 5$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \left[\frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12\right]$ cm^3

$= \left(4 \times \frac{22}{7} \times 25\right)$ cm^3

$= \left(\frac{2200}{7}\right)$ cm^3

Therefore, capacity of the conical vessel

$= \left(\frac{2200}{7000}\right)$ litres $\left(1 \text{ litre} = 1000 \text{ cm}^3\right)$

$= \frac{11}{35}$ litres

Question 3:

The height of a cone is 15 cm. If its volume is 1570 cm^3, find the diameter of its base. [Use π = 3.14]

Answer:

Height (h) of cone = 15 cm

Let the radius of the cone be r.

Volume of cone = 1570 cm^3

$\frac{1}{3}\pi r^2 h = 1570 \text{ cm}^3$

$\Rightarrow \left(\frac{1}{3} \times 3.14 \times r^2 \times 15\right) \text{ cm} = 1570 \text{ cm}^3$

$\Rightarrow r^2 = 100 \text{ cm}^2$

$\Rightarrow r = 10$ cm

Therefore, the radius of the base of cone is 10 cm.

Question 4:

If the volume of a right circular cone of height 9 cm is 48π cm^3, find the diameter of its base.

Answer:

Height (h) of cone = 9 cm

Let the radius of the cone be r.

Volume of cone = 48π cm^3

$$\Rightarrow \frac{1}{3}\pi r^2 h = 48\pi \text{ cm}^3$$

$$\Rightarrow \left(\frac{1}{3}\pi r^2 \times 9\right) \text{ cm} = 48\pi \text{ cm}^3$$

$$\Rightarrow r^2 = 16 \text{ cm}^2$$

$$\Rightarrow r = 4 \text{ cm}$$

Diameter of base = 2r = 8 cm

Question 5:

A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

Radius (r) of pit $= \left(\frac{3.5}{2}\right) \text{ m} = 1.75 \text{ m}$

Height (h) of pit = Depth of pit = 12 m

Volume of pit $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12\right] \text{ cm}^3$$

= 38.5 m^3

Thus, capacity of the pit = (38.5 × 1) kilolitres = 38.5 kilolitres

Question 6:

The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Radius of cone = $\left(\frac{28}{2}\right)\text{ cm} = 14\text{ cm}$

Let the height of the cone be h.

Volume of cone = 9856 cm^3

$$\Rightarrow \frac{1}{3}\pi r^2 h = 9856\text{ cm}^3$$

$$\Rightarrow \left[\frac{1}{3}\times\frac{22}{7}\times(14)^2\times h\right]\text{ cm}^2 = 9856\text{ cm}^3$$

h = 48 cm

Therefore, the height of the cone is 48 cm.

(ii) Slant height (l) of cone $= \sqrt{r^2 + h^2}$

$$= \left[\sqrt{(14)^2 + (48)^2}\right]\text{ cm}$$

$$= \left[\sqrt{196 + 2304}\right]\text{ cm}$$

$$= 50\text{ cm}$$

Therefore, the slant height of the cone is 50 cm.

(iii) CSA of cone = $\pi r l$

$$= \left(\frac{22}{7}\times 14\times 50\right)\text{ cm}^2$$

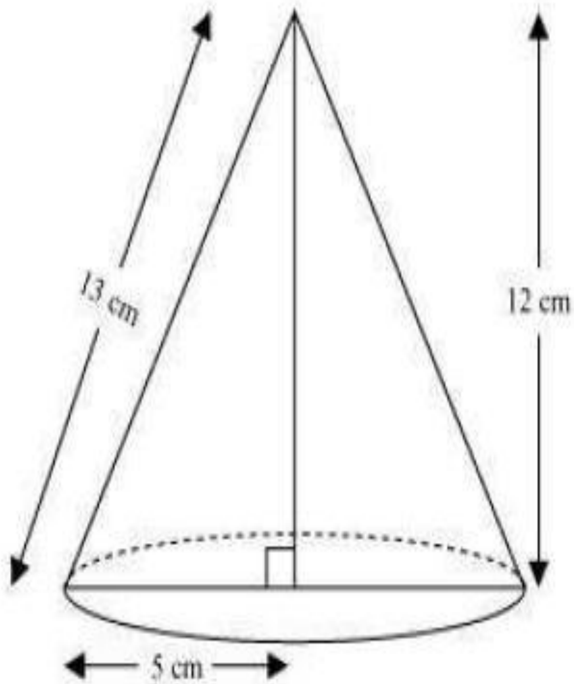
= 2200 cm^2

Therefore, the curved surface area of the cone is 2200 cm^2.

Question 7:

A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Answer:



When right-angled ΔABC is revolved about its side 12 cm, a cone with height (*h*) as 12 cm, radius (*r*) as 5 cm, and slant height (*l*) 13 cm will be formed.

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \left[\frac{1}{3} \times \pi \times (5)^2 \times 12\right] \text{ cm}^3$

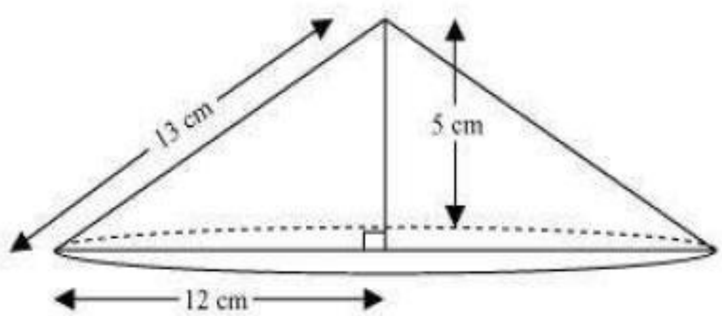
$= 100\pi \text{ cm}^3$

Therefore, the volume of the cone so formed is $100\pi \text{ cm}^3$.

Question 8:

If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Answer:



When right-angled ΔABC is revolved about its side 5 cm, a cone will be formed having radius (r) as 12 cm, height (h) as 5 cm, and slant height (l) as 13 cm.

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \left[\frac{1}{3} \times \pi \times (12)^2 \times 5\right] \text{cm}^3$

$= 240\pi \text{ cm}^3$

Therefore, the volume of the cone so formed is 240π cm^3.

Required ratio $= \frac{100\pi}{240\pi}$

$= \frac{5}{12} = 5:12$

Question 9:

A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of heap $= \left(\frac{10.5}{2}\right) \text{m} = 5.25 \text{ m}$

Height (h) of heap = 3 m

Volume of heap $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times (5.25)^2 \times 3 \right) \text{ m}^3$$

$$= 86.625 \text{ m}^3$$

Therefore, the volume of the heap of wheat is 86.625 m^3.

Area of canvas required = CSA of cone

$$= \pi rl = \pi r\sqrt{r^2 + h^2}$$

$$= \left[\frac{22}{7} \times 5.25 \times \sqrt{(5.25)^2 + (3)^2} \right] \text{ m}^2$$

$$= \left(\frac{22}{7} \times 5.25 \times 6.05 \right) \text{ m}^2$$

$$= 99.825 \text{ m}^2$$

Therefore, 99.825 m^2 canvas will be required to protect the heap from rain.

Exercise 13.8

Question 1:

Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Radius of sphere = 7 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \left[\frac{4}{3}\times\frac{22}{7}\times(7)^3\right] \text{ cm}^3$$

$$= \left(\frac{4312}{3}\right) \text{ cm}^3$$

$$= 1437\frac{1}{3} \text{ cm}^3$$

Therefore, the volume of the sphere is $1437\frac{1}{3}$ cm^3.

(ii) Radius of sphere = 0.63 m

Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \left[\frac{4}{3}\times\frac{22}{7}\times(0.63)^3\right] \text{ m}^3$$

$$= 1.0478 \text{ m}^3$$

Therefore, the volume of the sphere is 1.05 m^3 (approximately).

Question 2:

Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm (ii) 0.21 m

$\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

(i) Radius (r) of ball = $\left(\frac{28}{2}\right)\text{ cm} = 14\text{ cm}$

Volume of ball = $\frac{4}{3}\pi r^3$

$= \left[\frac{4}{3} \times \frac{22}{7} \times (14)^3\right]\text{ cm}^3$

$= 11498\frac{2}{3}\text{ cm}^3$

Therefore, the volume of the sphere is $11498\frac{2}{3}$ cm^3.

(ii)Radius (r) of ball = $\left(\frac{0.21}{2}\right)\text{ m}$ = 0.105 m

Volume of ball = $\frac{4}{3}\pi r^3$

$= \left[\frac{4}{3} \times \frac{22}{7} \times (0.105)^3\right]\text{ m}^3$

$= 0.004851\text{ m}^3$

Therefore, the volume of the sphere is 0.004851 m^3.

Question 3:

The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of metallic ball = $\left(\frac{4.2}{2}\right)\text{ cm} = 2.1\text{ cm}$

Volume of metallic ball = $\frac{4}{3}\pi r^3$

$$= \left[\frac{4}{3}\times\frac{22}{7}\times(2.1)^3\right] \text{cm}^3$$

$$= 38.808 \text{ cm}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Mass = Density × Volume

= (8.9 × 38.808) g

= 345.3912 g

Hence, the mass of the ball is 345.39 g (approximately).

Question 4:

The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Answer:

Let the diameter of earth be *d*. Therefore, the radius of earth will be $\frac{d}{2}$.

Diameter of moon will be $\frac{d}{4}$ and the radius of moon will be $\frac{d}{8}$.

Volume of moon = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{8}\right)^3 = \frac{1}{512}\times\frac{4}{3}\pi d^3$

Volume of earth = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{1}{8}\times\frac{4}{3}\pi d^3$

$$\frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{1}{512}\times\frac{4}{3}\pi d^3}{\frac{1}{8}\times\frac{4}{3}\pi d^3}$$

$$= \frac{1}{64}$$

$$\Rightarrow \text{Volume of moon} = \frac{1}{64}\text{Volume of earth}$$

Therefore, the volume of moon is $\frac{1}{64}$ of the volume of earth.

Question 5:

How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of hemispherical bowl = $\left(\frac{10.5}{2}\right)$ cm = 5.25 cm

Volume of hemispherical bowl = $\frac{2}{3}\pi r^3$

$$= \left[\frac{2}{3}\times\frac{22}{7}\times(5.25)^3\right] \text{ cm}^3$$

= 303.1875 cm^3

Capacity of the bowl = $\left(\frac{303.1875}{1000}\right)$ litre

$=0.3031875 \text{ litre} = 0.303 \text{ litre (approximately)}$

Therefore, the volume of the hemispherical bowl is 0.303 litre.

Question 6:

A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Inner radius (r_1) of hemispherical tank = 1 m

Thickness of hemispherical tank = 1 cm = 0.01 m

Outer radius (r_2) of hemispherical tank = (1 + 0.01) m = 1.01 m

$$\text{Volume of iron used to make such a tank} = \frac{2}{3}\pi\left(r_2^3 - r_1^3\right)$$

$$= \left[\frac{2}{3} \times \frac{22}{7} \times \left\{(1.01)^3 - (1)^3\right\}\right] \text{m}^3$$

$$= \left[\frac{44}{21} \times (1.030301 - 1)\right] \text{m}^3$$

$$= 0.06348 \text{ m}^3 \quad \text{(approximately)}$$

Question 7:

Find the volume of a sphere whose surface area is 154 cm^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let radius of sphere be r.

Surface area of sphere = 154 cm^2

$\Rightarrow 4\pi r^2 = 154$ cm^2

$$\Rightarrow r^2 = \left(\frac{154 \times 7}{4 \times 22}\right) \text{cm}^2$$

$$\Rightarrow r = \left(\frac{7}{2}\right) \text{cm} = 3.5 \text{ cm}$$

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \left[\frac{4}{3}\times\frac{22}{7}\times(3.5)^3\right]$ cm^3

$= 179\frac{2}{3}$ cm^3

Therefore, the volume of the sphere is $179\frac{2}{3}$ cm^3.

Question 8:

A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of Rs 498.96. If the cost of white-washing is Rs 2.00 per square meter, find the

(i) inside surface area of the dome,

(ii) volume of the air inside the dome. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

(i) Cost of white-washing the dome from inside = Rs 498.96

Cost of white-washing 1 m^2 area = Rs 2

Therefore, CSA of the inner side of dome = $\left(\frac{498.96}{2}\right)$ m^2

= 249.48 m^2

(ii) Let the inner radius of the hemispherical dome be *r*.

CSA of inner side of dome = 249.48 m^2

$2\pi r^2$ = 249.48 m^2

$\Rightarrow 2\times\frac{22}{7}\times r^2 = 249.48$ m^2

$\Rightarrow r^2 = \left(\frac{249.48\times 7}{2\times 22}\right)$ m$^2 = 39.69$ m^2

$\Rightarrow r = 6.3$ m

Volume of air inside the dome = Volume of hemispherical dome

$= \frac{2}{3}\pi r^3$

$= \left[\frac{2}{3} \times \frac{22}{7} \times (6.3)^3\right] \text{ m}^3$

= 523.908 m^3

= 523.9 m^3 (approximately)

Therefore, the volume of air inside the dome is 523.9 m^3.

Question 9:

Twenty seven solid iron spheres, each of radius *r* and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius *r'* of the new sphere, (ii) ratio of S and S'.

Answer:

(i)Radius of 1 solid iron sphere = *r*

Volume of 1 solid iron sphere $= \frac{4}{3}\pi r^3$

Volume of 27 solid iron spheres $= 27 \times \frac{4}{3}\pi r^3$

27 solid iron spheres are melted to form 1 iron sphere. Therefore, the volume of this iron sphere will be equal to the volume of 27 solid iron spheres. Let the radius of this new sphere be *r'*.

Volume of new solid iron sphere $= \frac{4}{3}\pi r'^3$

$\frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3$

$r'^3 = 27r^3$

$r' = 3r$

(ii) Surface area of 1 solid iron sphere of radius $r = 4\pi r^2$

Surface area of iron sphere of radius $r' = 4\pi (r')^2$

$= 4\pi (3r)^2 = 36\pi r^2$

$$\frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = 1:9$$

Question 10:

A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of capsule $= \left(\frac{3.5}{2}\right) \text{mm} = 1.75 \text{ mm}$

Volume of spherical capsule $= \frac{4}{3}\pi r^3$

$= \left[\frac{4}{3} \times \frac{22}{7} \times (1.75)^3\right] \text{mm}^3$

= 22.458 mm^3

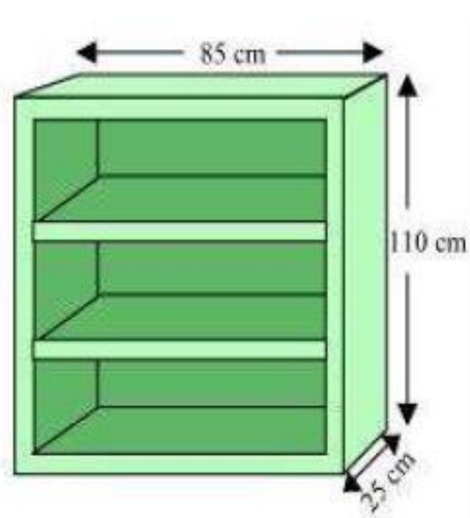
= 22.46 mm^3 (approximately)

Therefore, the volume of the spherical capsule is 22.46 mm^3.

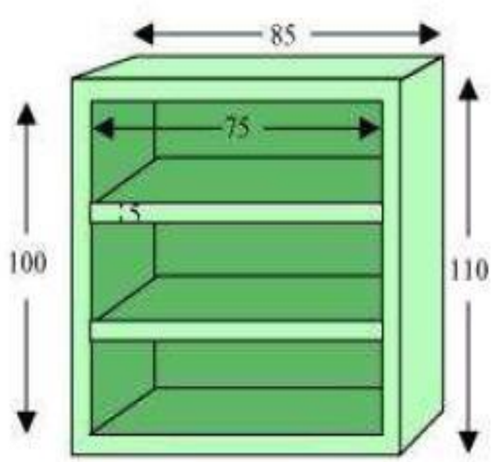
Exercise 13.9

Question 1:

A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see the given figure). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.



Answer:



External height (l) of book self = 85 cm

External breadth (b) of book self = 25 cm

External height (h) of book self = 110 cm

External surface area of shelf while leaving out the front face of the shelf

$= lh + 2\,(lb + bh)$

$= [85 \times 110 + 2\,(85 \times 25 + 25 \times 110)]\ \text{cm}^2$

$= (9350 + 9750)\ \text{cm}^2$

$= 19100\ \text{cm}^2$

Area of front face $= [85 \times 110 - 75 \times 100 + 2\,(75 \times 5)]\ \text{cm}^2$

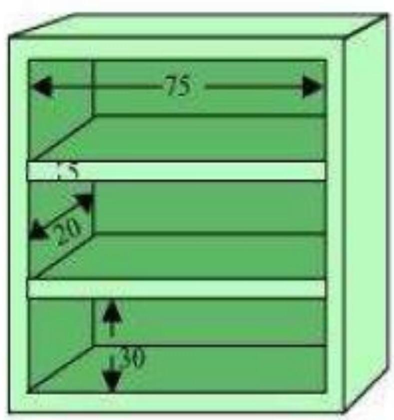
$= 1850 + 750\ \text{cm}^2$

$= 2600\ \text{cm}^2$

Area to be polished $= (19100 + 2600)\ \text{cm}^2 = 21700\ \text{cm}^2$

Cost of polishing 1 cm^2 area = Rs 0.20

Cost of polishing 21700 cm^2 area Rs (21700×0.20) = Rs 4340



It can be observed that length (l), breadth (b), and height (h) of each row of the book shelf is 75 cm, 20 cm, and 30 cm respectively.

Area to be painted in 1 row $= 2\,(l + h)\,b + lh$

$= [2\,(75 + 30) \times 20 + 75 \times 30]\ \text{cm}^2$

$= (4200 + 2250)\ \text{cm}^2$

$= 6450\ \text{cm}^2$

Area to be painted in 3 rows $= (3 \times 6450)\ \text{cm}^2 = 19350\ \text{cm}^2$

Cost of painting 1 cm^2 area = Rs 0.10

Cost of painting 19350 cm^2 area = Rs (19350×0.1)

= Rs 1935

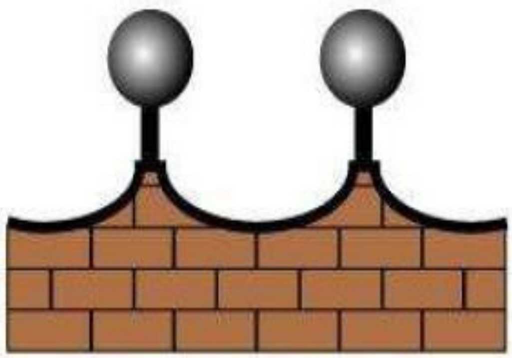
Total expense required for polishing and painting = Rs $(4340 + 1935)$

= Rs 6275

Therefore, it will cost Rs 6275 for polishing and painting the surface of the bookshelf.

Question 2:

The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in the given figure. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Answer:

Radius (r) of wooden sphere = $\left(\frac{21}{2}\right)$ cm $= 10.5$ cm

Surface area of wooden sphere = $4\pi r^2$

$$= \left[4 \times \frac{22}{7} \times (10.5)^2\right] \text{cm}^2 = 1386 \text{ cm}^2$$

Radius (r_1) of the circular end of cylindrical support = 1.5 cm

Height (h) of cylindrical support = 7 cm

CSA of cylindrical support = $2\pi rh$

$$= \left[2 \times \frac{22}{7} \times (1.5) \times 7\right] \text{cm}^2 = 66 \text{ cm}^2$$

Area of the circular end of cylindrical support = πr^2 $= \left[\frac{22}{7} \times (1.5)^2\right] \text{cm}^2$

= 7.07 cm^2

Area to be painted silver = [8 × (1386 − 7.07)] cm^2

= (8 × 1378.93) cm^2 = 11031.44 cm^2

Cost for painting with silver colour = Rs (11031.44 × 0.25) = Rs 2757.86

Area to be painted black = (8 × 66) cm^2 = 528 cm^2

Cost for painting with black colour = Rs (528 × 0.05) = Rs 26.40

Total cost in painting = Rs (2757.86 + 26.40)

= Rs 2784.26

Therefore, it will cost Rs 2784.26 in painting in such a way.

Question 3:

The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

Answer:

Let the diameter of the sphere be *d*.

Radius (r_1) of sphere $= \frac{d}{2}$

New radius (r_2) of sphere $= \frac{d}{2}\left(1 - \frac{25}{100}\right) = \frac{3}{8}d$

CSA (S_1) of sphere = $4\pi r_1^2$

$$= 4\pi\left(\frac{d}{2}\right)^2 = \pi d^2$$

CSA (S_2) of sphere when radius is decreased = $4\pi r_2^2$

$$= 4\pi\left(\frac{3d}{8}\right)^2 = \frac{9}{16}\pi d^2$$

Decrease in surface area of sphere = $S_1 - S_2$

$$= \pi d^2 - \frac{9}{16}\pi d^2$$

$$= \frac{7}{16}\pi d^2$$

Percentage decrease in surface area of sphere $= \frac{S_1 - S_2}{S_1} \times 100$

$$= \frac{7\pi d^2}{16\pi d^2} \times 100 = \frac{700}{16} = 43.75\%$$

NCERT
Class 9th Maths
Chapter 14: Statistics

Exercise 14.1

Question 1:

Give five examples of data that you can collect from day to day life.

Answer:

In our day to day life, we can collect the following data.

1. Number of females per 1000 males in various states of our country
2. Weights of students of our class
3. Production of wheat in the last 10 years in our country
4. Number of plants in our locality
5. Rainfall in our city in the last 10 years

Question 2:

Classify the data in Q1 above as primary or secondary data.

Answer:

The information which is collected by the investigator himself with a definite objective in his mind is called as primary data whereas when the information is gathered from a source which already had the information stored, it is called as secondary data. It can be observed that the data in 1, 3, and 5 is secondary data and the data in 2 and 4 is primary data.

Exercise 14.2

Question 1:

The blood groups of 30 students of Class VIII are recoded as follows:

A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,

A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.

Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students?

Answer:

It can be observed that 9 students have their blood group as A, 6 as B, 3 as AB, and 12 as O.

Therefore, the blood group of 30 students of the class can be represented as follows.

Blood group	**Number of students**
A	9
B	6
AB	3
O	12
Total	30

It can be observed clearly that the most common blood group and the rarest blood group among these students is O and AB respectively as 12 (maximum number of students) have their blood group as O, and 3 (minimum number of students) have their blood group as AB.

Question 3:

The relative humidity (in %) of a certain city for a month of 30 days was as follows:

98.1 98.6 99.2 90.3 86.5 95.3 92.9 96.3 94.2 95.1

89.2 92.3 97.1 93.5 92.7 95.1 97.2 93.3 95.2 97.3

96.2 92.1 84.9 90.2 95.7 98.3 97.3 96.1 92.1 89

(i) Construct a grouped frequency distribution table with classes

84 - 86, 86 - 88

(ii) Which month or season do you think this data is about?

(iii) What is the range of this data?

Answer:

(i) A grouped frequency distribution table of class size 2 has to be constructed. The class intervals will be 84 – 86, 86 – 88, and 88 – 90...

By observing the data given above, the required table can be constructed as follows.

Relative humidity (in %)	Number of days (frequency)
84 – 86	1
86 – 88	1
88 – 90	2
90 – 92	2
92 – 94	7
94 – 96	6
96 – 98	7
98 – 100	4
Total	30

(ii) It can be observed that the relative humidity is high. Therefore, the data is about a month of rainy season.

(iii) Range of data = Maximum value − Minimum value

= 99.2 − 84.9 = 14.3

Question 4:

The heights of 50 students, measured to the nearest centimeters, have been found to be as follows:

161 150 154 165 168 161 154 162 150 151

162 164 171 165 158 154 156 172 160 170

153 159 161 170 162 165 166 168 165 164

154 152 153 156 158 162 160 161 173 166

161 159 162 167 168 159 158 153 154 159

(i) Represent the data given above by a grouped frequency distribution table, taking the class intervals as 160 - 165, 165 - 170, etc.

(ii) What can you conclude bout their heights from the table?

Answer:

(i) A grouped frequency distribution table has to be constructed taking class intervals 160 − 165, 165 − 170, etc. By observing the data given above, the required table can be constructed as follows.

Height (in cm)	Number of students (frequency)
150 − 155	12
155 − 160	9
160− 165	14
165 − 170	10
170 − 175	5

Total	50

(ii) It can be concluded that more than 50% of the students are shorter than 165 cm.

Question 5:

A study was conducted to find out the concentration of sulphur dioxide in the air in parts per million (ppm) of a certain city. The data obtained for 30 days is as follows:

0.03 0.08 0.08 0.09 0.04 0.17

0.16 0.05 0.02 0.06 0.18 0.20

0.11 0.08 0.12 0.13 0.22 0.07

0.08 0.01 0.10 0.06 0.09 0.18

0.11 0.07 0.05 0.07 0.01 0.04

(i) Make a grouped frequency distribution table for this data with class intervals as 0.00 - 0.04, 0.04 - 0.08, and so on.

(ii) For how many days, was the concentration of sulphur dioxide more than 0.11 parts per million?

Answer:

Taking class intervals as 0.00, −0.04, 0.04, −0.08, and so on, a grouped frequency table can be constructed as follows.

Concentration of SO_2 (in ppm)	**Number of days (frequency)**
0.00 – 0.04	4
0.04 – 0.08	9
0.08 – 0.12	9
0.12 – 0.16	2
0.16 – 0.20	4

0.20 – 0.24	2
Total	30

The number of days for which the concentration of SO_2 is more than 0.11 is the number of days for which the concentration is in between 0.12 – 0.16, 0.16 – 0.20, 0.20 – 0.24.

Required number of days = 2 + 4 + 2 = 8

Therefore, for 8 days, the concentration of SO_2 is more than 0.11 ppm.

Question 6:

Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:

0 1 2 2 1 2 3 1 3 0

1 3 1 1 2 2 0 1 2 1

3 0 0 1 1 2 3 2 2 0

Prepare a frequency distribution table for the data given above.

Answer:

By observing the data given above, the required frequency distribution table can be constructed as follows.

Number of heads	**Number of times (frequency)**
0	6
1	10
2	9
3	5
Total	30

Question 7:

The value of π up to50 decimal places is given below:

3.14159265358979323846264338327950288419716939937510

(i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.

(ii) What are the most and the least frequently occurring digits?

Answer:

(i) By observation of the digits after decimal point, the required table can be constructed as follows.

Digit	Frequency
0	2
1	5
2	5
3	8
4	4
5	5
6	4
7	4
8	5
9	8
Total	50

(ii) It can be observed from the above table that the least frequency is 2 of digit 0, and the maximum frequency is 8 of digit 3 and 9. Therefore, the most frequently occurring digits are 3 and 9 and the least frequently occurring digit is 0.

Question 8:

Thirty children were asked about the number of hours they watched TV programmes in the previous week. The results were found as follows:

1 6 2 3 5 12 5 8 4 8

10 3 4 12 2 8 15 1 17 6

3 2 8 5 9 6 8 7 14 12

(i) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals as 5 - 10.

(ii) How many children watched television for 15 or more hours a week?

Answer:

(i) Our class intervals will be 0 − 5, 5 − 10, 10 −15…..

The grouped frequency distribution table can be constructed as follows.

Hours	**Number of children**
0 − 5	10
5 − 10	13
10 − 15	5
15 − 20	2
Total	30

(ii) The number of children who watched TV for 15 or more hours a week is 2 (i.e., the number of children in class interval 15 − 20).

Question 9:

A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows:

2.6 3.0 3.7 3.2 2.2 4.1 3.5 4.5

3.5 2.3 3.2 3.4 3.8 3.2 4.6 3.7

2.5 4.4 3.4 3.3 2.9 3.0 4.3 2.8

3.5 3.2 3.9 3.2 3.2 3.1 3.7 3.4

4.6 3.8 3.2 2.6 3.5 4.2 2.9 3.6

Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from the intervals 2 – 2.5.

Answer:

A grouped frequency table of class size 0.5 has to be constructed, starting from class interval 2 – 2.5.

Therefore, the class intervals will be 2 – 2.5, 2.5 – 3, 3 – 3.5...

By observing the data given above, the required grouped frequency distribution table can be constructed as follows.

Lives of batteries (in hours)	**Number of batteries**
2 – 2.5	2
2.5 – 3.0	6
3.0 – 3.5	14
3.5– 4.0	11
4.0 – 4.5	4
4.5 – 5.0	3
Total	40

Exercise 14.3

Question 1:

A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 – 44 (in years) worldwide, found the following figures (in %):

S.No.	**Causes**	**Female fatality rate (%)**
1.	Reproductive health conditions	31.8
2.	Neuropsychiatric conditions	25.4
3.	Injuries	12.4
4.	Cardiovascular conditions	4.3
5.	Respiratory conditions	4.1
6.	Other causes	22.0

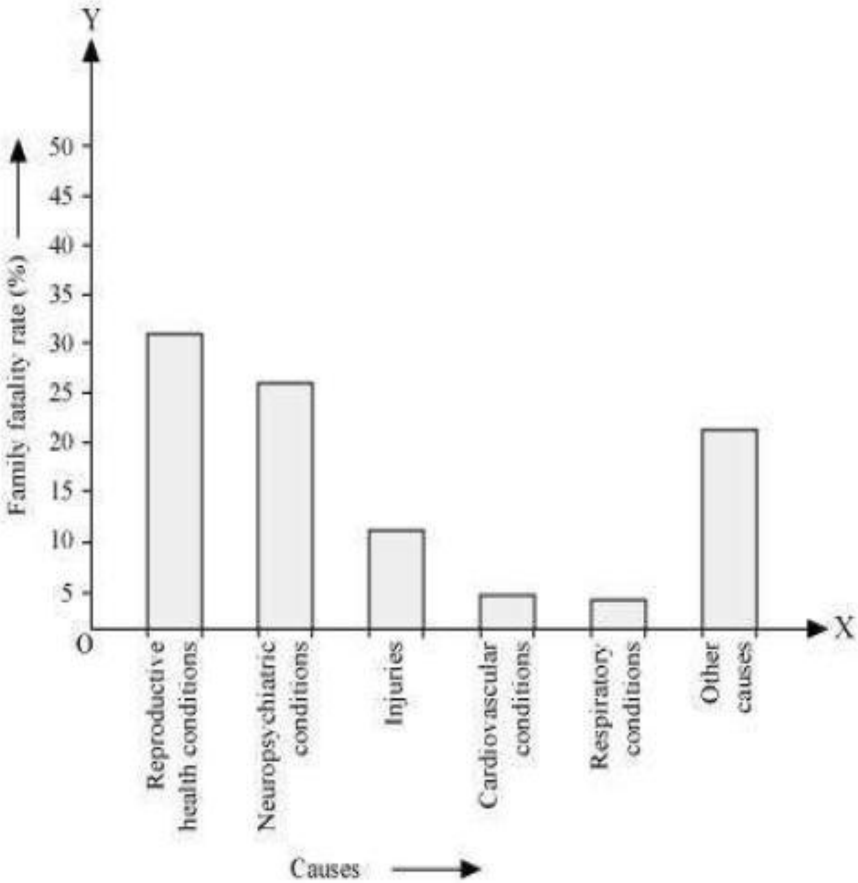
(i) Represent the information given above graphically.

(ii) Which condition is the major cause of women's ill health and death worldwide?

(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

Answer:

(i) By representing causes on x-axis and family fatality rate on y-axis and choosing an appropriate scale (1 unit = 5% for y axis), the graph of the information given above can be constructed as follows.



All the rectangle bars are of the same width and have equal spacing between them.

(ii) Reproductive health condition is the major cause of women's ill health and death worldwide as 31.8% of women are affected by it.

(iii) The factors are as follows.

1. Lack of medical facilities

2. Lack of correct knowledge of treatment

Question 2:

The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below.

Section	Number of girls per thousand boys
Scheduled Caste (SC)	940
Scheduled Tribe (ST)	970

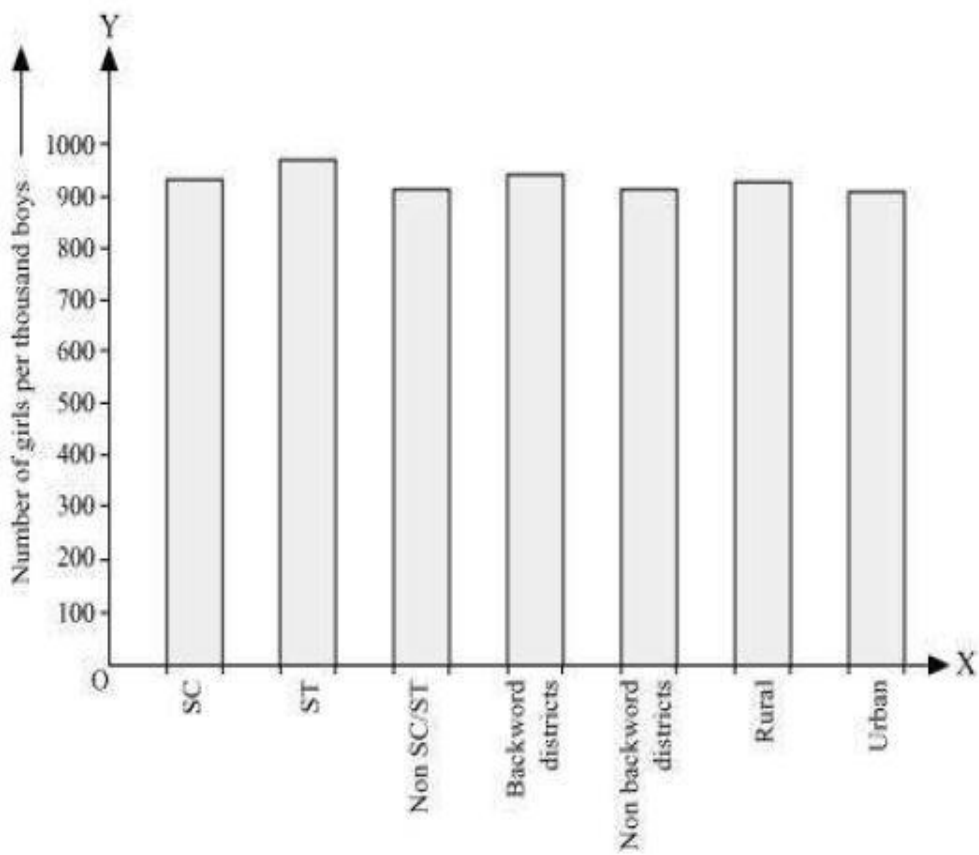
Non SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.

(ii) In the classroom discuss what conclusions can be arrived at from the graph.

Answer:

(i) By representing section (variable) on x-axis and number of girls per thousand boys on y-axis, the graph of the information given above can be constructed by choosing an appropriate scale (1 unit = 100 girls for *y*-axis)



Here, all the rectangle bars are of the same length and have equal spacing in between them.

(ii) It can be observed that maximum number of girls per thousand boys (i.e., 970) is for ST and minimum number of girls per thousand boys (i.e., 910) is for urban.

Also, the number of girls per thousand boys is greater in rural areas than that in urban areas, backward districts than that in non-backward districts, SC and ST than that in non-SC/ST.

Question 3:

Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

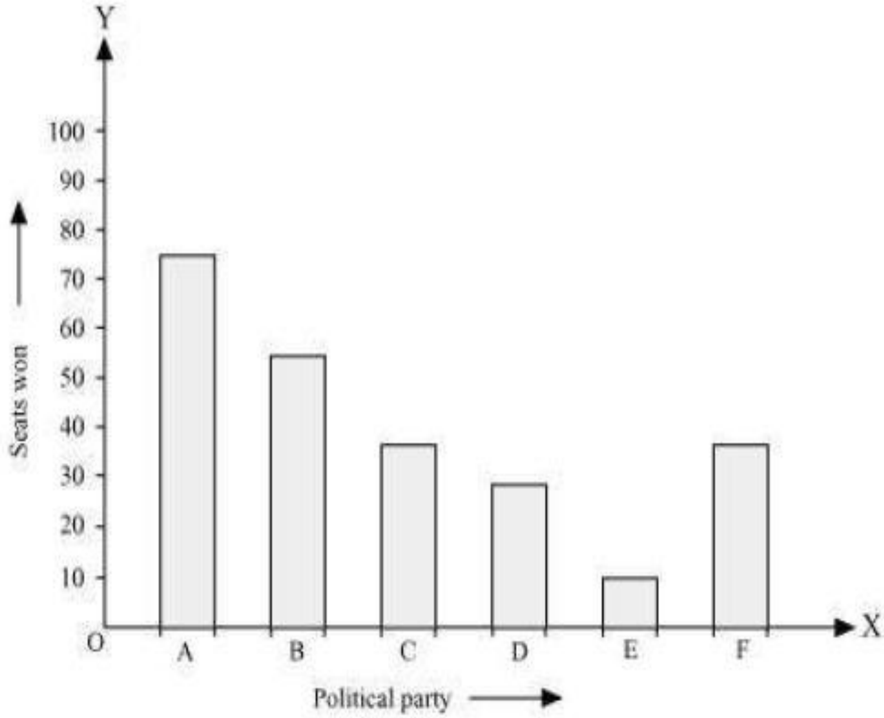
Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

(i) Draw a bar graph to represent the polling results.

(ii) Which political party won the maximum number of seats?

Answer:

(i) By taking polling results on *x*-axis and seats won as *y*-axis and choosing an appropriate scale (1 unit = 10 seats for *y*-axis), the required graph of the above information can be constructed as follows.



Here, the rectangle bars are of the same length and have equal spacing in between them.

(ii) Political party 'A' won maximum number of seats.

Question 4:

The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following table:

Length (in mm)	Number of leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

(i) Draw a histogram to represent the given data.

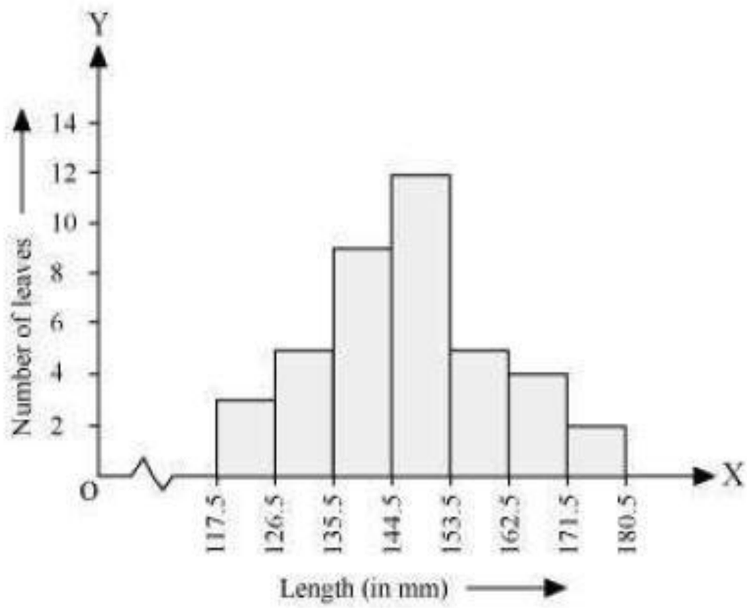
(ii) Is there any other suitable graphical representation for the same data?

(iii) Is it correct to conclude that the maximum number of leaves are 153 mm long? Why?

Answer:

(i) It can be observed that the length of leaves is represented in a discontinuous class interval having a difference of 1 in between them. Therefore, $\frac{1}{2} = 0.5$ has to be added to each upper class limit and also have to subtract 0.5 from the lower class limits so as to make the class intervals continuous.

Length (in mm)	**Number of leaves**
117.5 – 126.5	3
126.5 – 135.5	5
135.5 – 144.5	9
144.5 – 153.5	12
153.5 – 162.5	5
162.5 – 171.5	4
171.5 – 180.5	2



Taking the length of leaves on *x*-axis and the number of leaves on *y*-axis, the histogram of this information can be drawn as above.

Here, 1 unit on *y*-axis represents 2 leaves.

(ii) Other suitable graphical representation of this data is frequency polygon.

(iii) No, as maximum number of leaves (i.e., 12) has their length in between 144.5 mm and 153.5 mm. It is not necessary that all have their lengths as 153 mm.

Question 5:

The following table gives the life times of neon lamps:

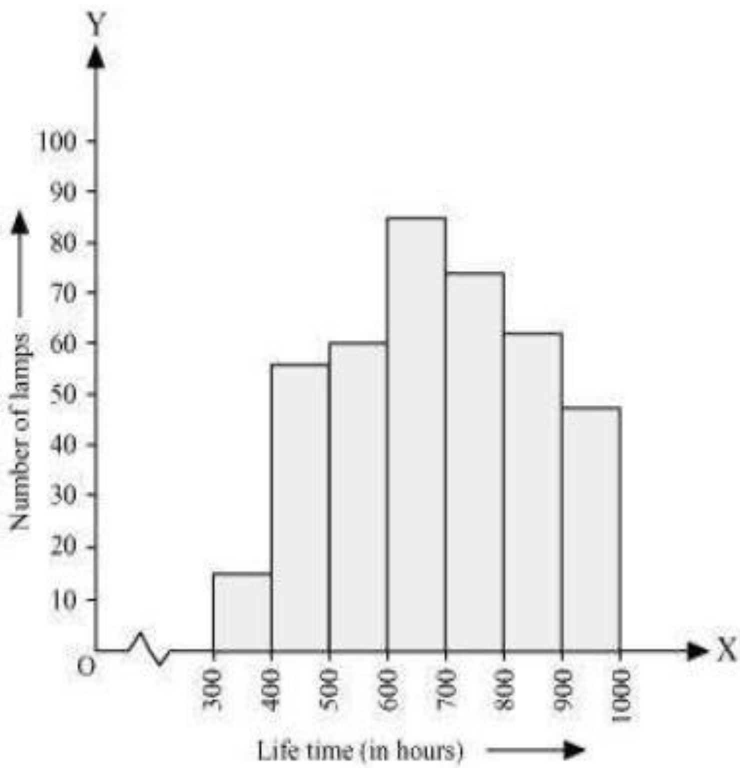
Length (in hours)	Number of lamps
300 – 400	14
400 – 500	56
500 – 600	60
600 – 700	86
700 – 800	74
800 – 900	62
900 – 1000	48

(i) Represent the given information with the help of a histogram.

(ii) How many lamps have a lifetime of more than 700 hours?

Answer:

(i) By taking life time (in hours) of neon lamps on *x*-axis and the number of lamps on *y*-axis, the histogram of the given information can be drawn as follows.



Here, 1 unit on *y*-axis represents 10 lamps.

(ii) It can be concluded that the number of neon lamps having their lifetime more than 700 is the sum of the number of neon lamps having their lifetime as 700 – 800, 800 – 900, and 900 – 1000.

Therefore, the number of neon lamps having their lifetime more than 700 hours is 184. (74 + 62 + 48 = 184)

Question 6:

The following table gives the distribution of students of two sections according to the mark obtained by them:

Section A		**Section B**	
Marks	**Frequency**	**Marks**	**Frequency**

0 − 10	3	0 − 10	5
10 − 20	9	10 − 20	19
20 − 30	17	20 − 30	15
30 − 40	12	30 − 40	10
40 − 50	9	40 − 50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

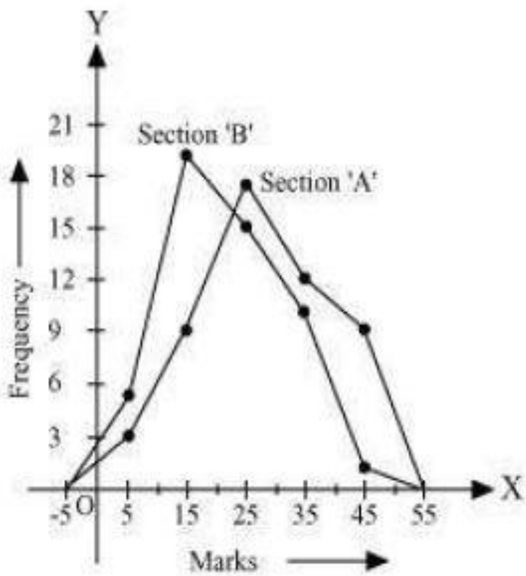
Answer:

We can find the class marks of the given class intervals by using the following formula.

Class mark $= \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$

Section A			Section B		
Marks	**Class marks**	**Frequency**	**Marks**	**Class marks**	**Frequency**
0 − 10	5	3	0 − 10	5	5
10 − 20	15	9	10 − 20	15	19
20 − 30	25	17	20 − 30	25	15
30 − 40	35	12	30 − 40	35	10
40 − 50	45	9	40 − 50	45	1

Taking class marks on *x*-axis and frequency on *y*-axis and choosing an appropriate scale (1 unit = 3 for *y*-axis), the frequency polygon can be drawn as follows.



It can be observed that the performance of students of section 'A' is better than the students of section 'B' in terms of good marks.

Question 7:

The runs scored by two teams A and B on the first 60 balls in a cricket match are given below:

Number of balls	Team A	Team B
1 – 6	2	5
7 – 12	1	6
13 – 18	8	2
19 – 24	9	10
25 – 30	4	5
31 – 36	5	6
37 – 42	6	3
43 – 48	10	4
49 – 54	6	8
55 – 60	2	10

Represent the data of both the teams on the same graph by frequency polygons.

[**Hint:** First make the class intervals continuous.]

Answer:

It can be observed that the class intervals of the given data are not continuous. There is a gap of 1 in between them. Therefore, $\frac{1}{2}=0.5$ has to be added to the upper class limits and 0.5 has to be subtracted from the lower class limits.

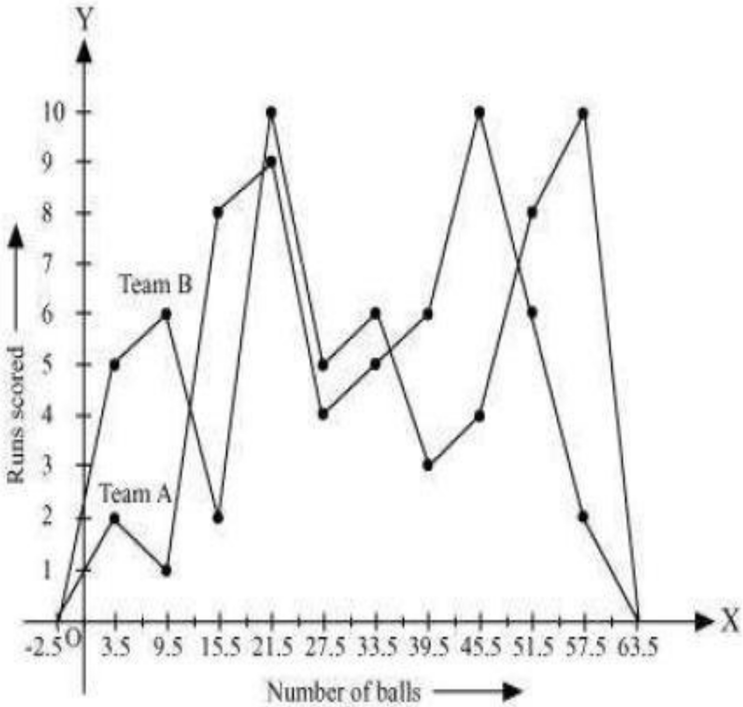
Also, class mark of each interval can be found by using the following formula.

Class mark $=\frac{\text{Upper class limit } + \text{ Lower class limit}}{2}$

Continuous data with class mark of each class interval can be represented as follows.

Number of balls	**Class mark**	**Team A**	**Team B**
0.5 – 6.5	3.5	2	5
6.5 – 12.5	9.5	1	6
12.5 – 18.5	15.5	8	2
18.5 – 24.5	21.5	9	10
24.5 – 30.5	27.5	4	5
30.5 – 36.5	33.5	5	6
36.5 – 42.5	39.5	6	3
42.5 – 48.5	45.5	10	4
48.5 – 54.5	51.5	6	8
54.5 – 60.5	57.5	2	10

By taking class marks on *x*-axis and runs scored on *y*-axis, a frequency polygon can be constructed as follows.



Question 8:

A random survey of the number of children of various age groups playing in park was found as follows:

Age (in years)	Number of children
1 − 2	5
2 − 3	3
3 − 5	6
5 − 7	12
7 − 10	9
10 − 15	10
15 − 17	4

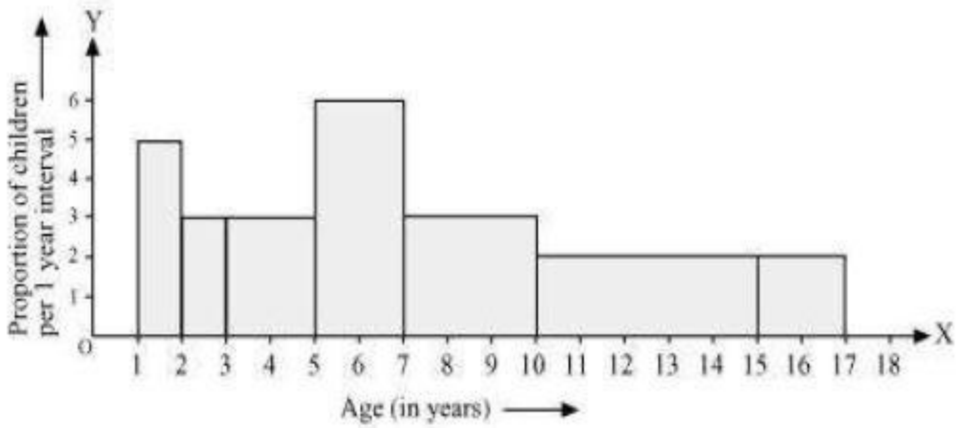
Draw a histogram to represent the data above.

Answer:

Here, it can be observed that the data has class intervals of varying width. The proportion of children per 1 year interval can be calculated as follows.

Age (in years)	Frequency (Number of children)	Width of class	Length of rectangle
1 – 2	5	1	$\frac{5\times1}{1}=5$
2 – 3	3	1	$\frac{3\times1}{1}=3$
3 – 5	6	2	$\frac{6\times1}{2}=3$
5 – 7	12	2	$\frac{12\times1}{2}=6$
7 – 10	9	3	$\frac{9\times1}{3}=3$
10 – 15	10	5	$\frac{10\times1}{5}=2$
15 – 17	4	2	$\frac{4\times1}{2}=2$

Taking the age of children on x-axis and proportion of children per 1 year interval on y-axis, the histogram can be drawn as follows.



Question 9:

100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:

Number of letters	Number of surnames
1 – 4	6
4 – 6	30
6 – 8	44
8 – 12	16
12 – 20	4

(i) Draw a histogram to depict the given information.

(ii) Write the class interval in which the maximum number of surname lie.

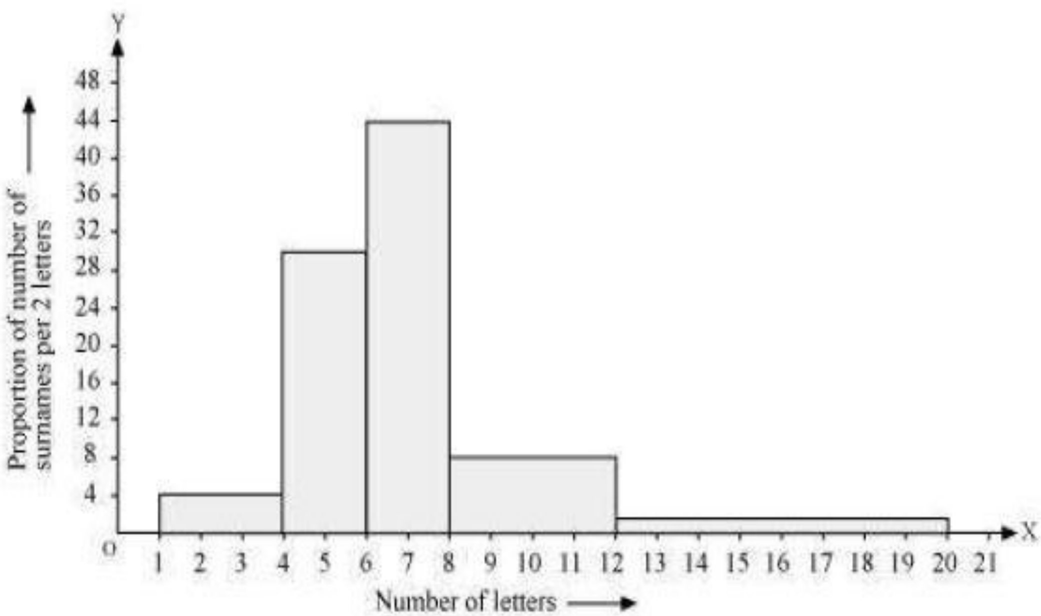
Answer:

(i) Here, it can be observed that the data has class intervals of varying width. The proportion of the number of surnames per 2 letters interval can be calculated as follows.

Number of letters	Frequency (Number of surnames)	Width of class	Length of rectangle

1 – 4	6	3	$\frac{6\times2}{3}=4$
4 – 6	30	2	$\frac{30\times2}{2}=30$
6 – 8	44	2	$\frac{44\times2}{2}=44$
8 – 12	16	4	$\frac{16\times2}{4}=8$
12 – 20	4	8	$\frac{4\times2}{8}=1$

By taking the number of letters on *x*-axis and the proportion of the number of surnames per 2 letters interval on *y*-axis and choosing an appropriate scale (1 unit = 4 students for *y* axis), the histogram can be constructed as follows.



(ii) The class interval in which the maximum number of surnames lies is 6 – 8 as it has 44 surnames in it i.e., the maximum for this data.

Exercise 14.4

Question 1:

The following number of goals was scored by a team in a series of 10 matches:

2, 3, 4, 5, 0, 1, 3, 3, 4, 3

Find the mean, median and mode of these scores.

Answer:

The number of goals scored by the team is

2, 3, 4, 5, 0, 1, 3, 3, 4, 3

$$\text{Mean of data} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

$$\text{Mean score} = \frac{2+3+4+5+0+1+3+3+4+3}{10}$$

$$= \frac{28}{10} = 2.8$$

$$= 2.8 \text{ goals}$$

Arranging the number of goals in ascending order,

0, 1, 2, 3, 3, 3, 3, 4, 4, 5

The number of observations is 10, which is an even number. Therefore, median score will be the mean of $\frac{10}{2}$ i.e., 5^{th} and $\frac{10}{2}+1$ i.e., 6^{th} observation while arranged in ascending or descending order.

$$\text{Median score} = \frac{5^{th} \text{ observation} + 6^{th} \text{ observation}}{2}$$

$$= \frac{3+3}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

Mode of data is the observation with the maximum frequency in data.

Therefore, the mode score of data is 3 as it has the maximum frequency as 4 in the data.

Question 2:

In a mathematics test given to 15 students, the following marks (out of 100) are recorded:

41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60

Find the mean, median and mode of this data.

Answer:

The marks of 15 students in mathematics test are

41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60

$$\text{Mean of data} = \frac{\text{Sum of all observation}}{\text{Total number of observation}}$$

$$= \frac{41+39+48+52+46+62+54+40+96+52+98+40+42+52+60}{15}$$

$$= \frac{822}{15} = 54.8$$

Arranging the scores obtained by 15 students in an ascending order,

39, 40, 40, 41, 42, 46, 48, 52, 52, 52, 54, 60, 62, 96, 98

As the number of observations is 15 which is odd, therefore, the median of data will be $\frac{15+1}{2}$ = 8^{th} observation whether the data is arranged in an ascending or descending order.

Therefore, median score of data = 52

Mode of data is the observation with the maximum frequency in data. Therefore, mode of this data is 52 having the highest frequency in data as 3.

Question 3:

The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x.

29, 32, 48, 50, x, x + 2, 72, 78, 84, 95

Answer:

It can be observed that the total number of observations in the given data is 10 (even number). Therefore, the median of this data will be the mean of $\frac{10}{2}$ i.e., 5th and $\frac{10}{2}+1$ i.e., 6th observation.

$$\text{Therefore, median of data} = \frac{5^{th}\text{ observation} + 6^{th}\text{ observation}}{2}$$

$$\Rightarrow 63 = \frac{x+x+2}{2}$$

$$\Rightarrow 63 = \frac{2x+2}{2}$$

$$\Rightarrow 63 = x+1$$

$$\Rightarrow x = 62$$

Question 4:

Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

Answer:

Arranging the data in an ascending order,

14, 14, 14, 14, 17, 18, 18, 18, 22, 23, 25, 28

It can be observed that 14 has the highest frequency, i.e. 4, in the given data. Therefore, mode of the given data is 14.

Question 5:

Find the mean salary of 60 workers of a factory from the following table:

Salary (in Rs)	Number of workers
3000	16
4000	12
5000	10

6000	8
7000	6
8000	4
9000	3
1000	1
Total	60

Answer:

We know that

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

The value of $\sum f_i x_i$ and $\sum f_i$ can be calculated as follows.

Salary (in Rs) (x_i)	**Number of workers (f_i)**	$f_i x_i$
3000	16	3000 × 16 = 48000
4000	12	4000 × 12 = 48000
5000	10	5000 × 10 = 50000
6000	8	6000 × 8 = 48000
7000	6	7000 × 6 = 42000
8000	4	8000 × 4 = 32000
9000	3	9000 × 3 = 27000

10000	1	$10000 \times 1 = 10000$
Total	$\sum f_i = 60$	$\sum f_i x_i = 305000$

$$\text{Mean salary} = \frac{305000}{60}$$
$$= 5083.33$$

Therefore, mean salary of 60 workers is Rs 5083.33.

Question 6:

Give one example of a situation in which

(i) The mean is an appropriate measure of central tendency.

(ii) The mean is not an appropriate measure of central tendency but the median is an appropriate measure of central tendency.

Answer:

When any data has a few observations such that these are very far from the other observations in it, it is better to calculate the median than the mean of the data as median gives a better estimate of average in this case.

(i) Consider the following example – the following data represents the heights of the members of a family.

154.9 cm, 162.8 cm, 170.6 cm, 158.8 cm, 163.3 cm, 166.8 cm, 160.2 cm

In this case, it can be observed that the observations in the given data are close to each other. Therefore, mean will be calculated as an appropriate measure of central tendency.

(ii) The following data represents the marks obtained by 12 students in a test.

48, 59, 46, 52, 54, 46, 97, 42, 49, 58, 60, 99

In this case, it can be observed that there are some observations which are very far from other observations. Therefore, here, median will be calculated as an appropriate measure of central tendency.

NCERT

Class 9th Maths

Chapter 15: Probability

Exercise 15.1

Question 1:

In a cricket math, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

Answer:

Number of times the batswoman hits a boundary = 6

Total number of balls played = 30

∴ Number of times that the batswoman does not hit a boundary = 30 − 6 = 24

$$P\text{ (she does not hit a boundary)} = \frac{\text{Number of times when she does not hit boundary}}{\text{Total number of balls played}}$$

$$= \frac{24}{30} = \frac{4}{5}$$

Question 2:

1500 families with 2 children were selected randomly, and the following data were recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

(i) 2 girls (ii) 1 girl (iii) No girl

Also check whether the sum of these probabilities is 1.

Answer:

Total number of families = 475 + 814 + 211

= 1500

(i) Number of families having 2 girls = 475

$$P_1\text{ (a randomly chosen family has 2 girls)} = \frac{\text{Number of families having 2 girls}}{\text{Total number of families}}$$

$$= \frac{475}{1500} = \frac{19}{60}$$

(ii) Number of families having 1 girl = 814

$$P_2 \text{ (a randomly chosen family has 1 girl)} = \frac{\text{Number of families having 1 girl}}{\text{Total number of families}}$$

$$= \frac{814}{1500} = \frac{407}{750}$$

(iii) Number of families having no girl = 211

$$P_3 \text{ (a randomly chosen family has no girl)} = \frac{\text{Number of families having no girl}}{\text{Total number of families}}$$

$$= \frac{211}{1500}$$

$$\text{Sum of all these probabilities} = \frac{19}{60} + \frac{407}{750} + \frac{211}{1500}$$

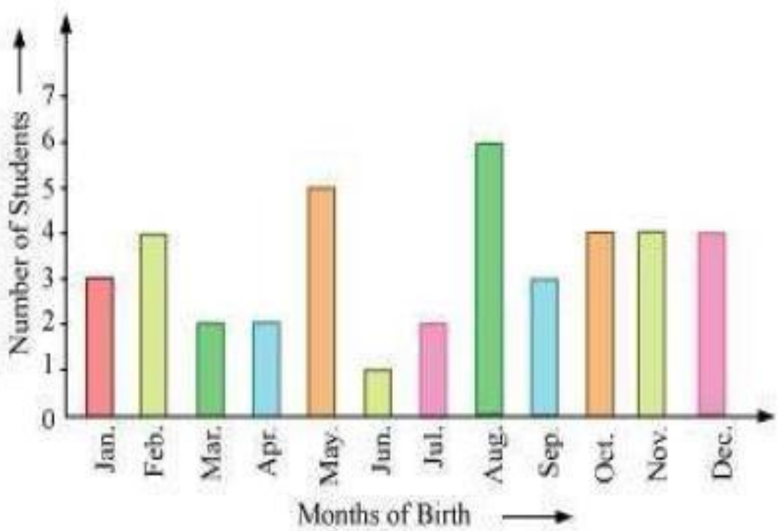
$$= \frac{475 + 814 + 211}{1500}$$

$$= \frac{1500}{1500} = 1$$

Therefore, the sum of all these probabilities is 1.

Question 3:

In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained:



Find the probability that a student of the class was born in August.

Answer:

Number of students born in the month of August = 6

Total number of students = 40

$$P\text{ (Students born in the month of August)} = \frac{\text{Number of students born in August}}{\text{Total number of students}}$$

$$= \frac{6}{40} = \frac{3}{20}$$

Question 4:

Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

Answer:

Number of times 2 heads come up = 72

Total number of times the coins were tossed = 200

$$P(2\text{ heads will come up}) = \frac{\text{Number of times 2 heads come up}}{\text{Total number of times the coins were tossed}}$$

$$= \frac{72}{200} = \frac{9}{25}$$

Question 5:

An organization selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

Monthly income (in Rs)	**Vehicles per family**			
	0	**1**	**2**	**Above 2**

Less than 7000	10	160	25	0
7000 – 10000	0	305	27	2
10000 – 13000	1	535	29	1
13000 – 16000	2	469	59	25
16000 or more	1	579	82	88

Suppose a family is chosen, find the probability that the family chosen is

(i) earning Rs 10000 – 13000 per month and owning exactly 2 vehicles.

(ii) earning Rs 16000 or more per month and owning exactly 1 vehicle.

(iii) earning less than Rs 7000 per month and does not own any vehicle.

(iv) earning Rs 13000 – 16000 per month and owning more than 2 vehicles.

(v) owning not more than 1 vehicle.

Answer:

Number of total families surveyed = 10 + 160 + 25 + 0 + 0 + 305 + 27 + 2 + 1 + 535 + 29 + 1 + 2 + 469 + 59 + 25 + 1 + 579 + 82 + 88 = 2400

(i) Number of families earning Rs 10000 – 13000 per month and owning exactly 2 vehicles = 29

Hence, required probability, $P = \frac{29}{2400}$

(ii) Number of families earning Rs 16000 or more per month and owning exactly 1 vehicle = 579

Hence, required probability, $P = \frac{579}{2400}$

(iii) Number of families earning less than Rs 7000 per month and does not own any vehicle = 10

Hence, required probability, $P = \frac{10}{2400} = \frac{1}{240}$

(iv) Number of families earning Rs 13000 − 16000 per month and owning more than 2 vehicles = 25

Hence, required probability, $P = \frac{25}{2400} = \frac{1}{96}$

(v) Number of families owning not more than 1 vehicle = 10 + 160 + 0 + 305 + 1 + 535 + 2 + 469 + 1 + 579 = 2062

Hence, required probability, $P = \frac{2062}{2400} = \frac{1031}{1200}$

Question 6:

A teacher wanted to analyse the performance of two sections of students in a mathematics test of 100 marks. Looking at their performances, she found that a few students got under 20 marks and a few got 70 marks or above. So she decided to group them into intervals of varying sizes as follows: 0 − 20, 20 − 30... 60 − 70, 70 − 100. Then she formed the following table:

Marks	**Number of student**
0 − 20	7
20 − 30	10
30 − 40	10
40 − 50	20
50 − 60	20
60 − 70	15
70 − above	8
Total	90

(i) Find the probability that a student obtained less than 20 % in the mathematics test.

(ii) Find the probability that a student obtained marks 60 or above.

Answer:

Totalnumber of students = 90

(i) Number of students getting less than 20 % marks in the test = 7

Hence, required probability, $P = \frac{7}{90}$

(ii) Number of students obtaining marks 60 or above = 15 + 8 = 23

Hence, required probability, $P = \frac{23}{90}$

Question 7:

To know the opinion of the students about the subject *statistics*, a survey of 200 students was conducted. The data is recorded in the following table.

Opinion	Number of students
like	135
dislike	65

Find the probability that a student chosen at random

(i) likes statistics, (ii) does not like it

Answer:

Total number of students = 135 + 65 = 200

(i) Number of students liking statistics = 135

$$P(\text{students liking statistics}) = \frac{135}{200} = \frac{27}{40}$$

(ii) Number of students who do not like statistics = 65

$$P(\text{students not liking statistics}) = \frac{65}{200} = \frac{13}{40}$$

Question 8:

The distance (in km) of 40 engineers from their residence to their place of work were found as follows.

5 3 10 20 25 11 13 7 12 31

19 10 12 17 18 11 32 17 16 2

7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

What is the empirical probability that an engineer lives:

(i) less than 7 km from her place of work?

(ii) more than or equal to 7 km from her place of work?

(iii) within $\frac{1}{2}$ km from her place of work?

Answer:

(i) Total number of engineers = 40

Number of engineers living less than 7 km from their place of work = 9

Hence, required probability that an engineer lives less than 7 km from her place of work, $P = \frac{9}{40}$

(ii) Number of engineers living more than or equal to 7 km from their place of work = 40 − 9 = 31

Hence, required probability that an engineer lives more than or equal to 7 km from her place of work, $P = \frac{31}{40}$

(iii) Number of engineers living within $\frac{1}{2}$ km from her place of work = 0

Hence, required probability that an engineer lives within $\frac{1}{2}$ km from her place of work, P = 0

Question 11:

Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):

4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00

Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

Answer:

Number of total bags = 11

Number of bags containing more than 5 kg of flour = 7

Hence, required probability, $P = \frac{7}{11}$

Question 12:

Concentration of SO_2 (in ppm)	**Number of days (frequency)**
0.00 – 0.04	4
0.04 – 0.08	9
0.08 – 0.12	9
0.12 – 0.16	2
0.16 – 0.20	4
0.20 – 0.24	2
Total	30

The above frequency distribution table represents the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 – 0.16 on any of these days.

Answer:

Number days for which the concentration of sulphur dioxide was in the interval of 0.12 – 0.16 = 2

Total number of days = 30

Hence, required probability, $P = \frac{2}{30} = \frac{1}{15}$

Question 13:

Blood group	Number of students
A	9
B	6
AB	3
O	12
Total	30

The above frequency distribution table represents the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class, selected at random, has blood group AB.

Answer:

Number of students having blood group AB = 3

Total number of students = 30

Hence, required probability, $P = \frac{3}{30} = \frac{1}{10}$